We develop a stochastic model of electoral competition in order to study the economic and political determinants of trade policy. We model a small open economy with two tradable goods, each of which is produced using a sector specific factor (e.g., land and capital) and another factor that is mobile between these tradable sectors (labor); one nontradable good, which is also produced using a specific factor (skilled labor), and an elected government with the mandate to tax trade flows. The tax revenue is used to provide local public goods that increase the economic agents’ utility. We use this general equilibrium model to explicitly derive the preferences of the different socioeconomic groups in society (landlords, industrialists, labor and skilled workers). We then use those derived preferences for policies to model the individual probabilistic voting behavior of the members of each of these socioeconomic groups. We use this model to shed light on how differences in the comparative advantages of countries explain trade policy divergence between countries as well as trade policy instability within countries. We regard trade policy instability to mean that, in equilibrium, political parties diverge in terms of the political platforms they adopt. We show that in natural resource (land) abundant economies with very little capital, or in economies that specializes in the production of manufactures, parties tend to converge to the same policy platform, and trade policy is likely to be stable and relatively close to free trade. In contrast, in a natural resource abundant economy with an important domestic industry that competes with the imports, parties tend to diverge, and trade policy is likely to be more protectionist and unstable.

Keywords: Trade policy; electoral competition; policy divergence; and trade policy stability.

*We appreciate very helpful comments from Daniel Heymann and Paulo Somaini. Norman Schofield acknowledges financial support from NSF grant 0715929 and Sebastian Galiani acknowledges financial support from ESRC grant 062-23-1360.

1Corresponding author: galiani@wustl.edu
2schofield.norman@gmail.com
3gftorren@artscl.wustl.edu
1 Introduction

Many developing countries adopted trade protectionist measures during the second part of the twentieth century. Most of these countries, if not all of them, did not have a comparative advantage in the manufacturing sector and they did not industrialize in a sustainable way as a result. Instead, they had a comparative advantage within the primary sector.\footnote{See Syrquin (1989).} In contrast, countries with comparative advantage in the manufacturing sector tended to remain much more open to trade. Additionally, the countries that adopted import substitution policies tended to show substantial volatility over time in their trade policies. Consider, just as an example, the case of Argentina. This country is relatively well endowed with highly productive land, and its comparative advantage has always been in the production of primary goods.\footnote{See Brambilla, Galiani and Porto (2009).} Up to the 1930s, Argentina was well integrated to the world economy, and though some protectionism naturally developed during the world recession of the 1930s, only after World War II, when for the first time workers massively voted in a presidential election\footnote{See Cantón (1968).}, the country closed itself off in large degree from world markets becoming almost autarkic until the mid-1970s. Since then, though the country tended to reintegrate with the world economy, trade policies were highly volatile. Moreover, Hopenhayn and Neumeyer (2005) argue that this uncertainty about trade policy significantly hampered capital accumulation during this period.\footnote{See also Persson and Tabellini (2006).}

This brief sketch is meant to suggest the close and complex connections between political choice and economic structure. Many models of political choice emphasize political convergence to an electoral mean or median.\footnote{There is an enormous literature on this topic. See for example Downs (1957), Hinich (1977), Ledyard (1981, 1984), Enelow and Hinich (1982, 1984, 1989), Coughlin (1992), Lin, Enelow, and Dorussen (1999), Banks and Duggan, (2005), McKelvey and Patty (2006).} Such models appear to be of limited use in explaining the oscillations that can occur as a result of divergent political choices by parties. Recent developments of the spatial stochastic model suggest, however, that political parties will not converge if there is sufficient difference in the valences of political leaders. Schofield and Sened (2006), following Stokes (1963, 1992), give this definition of valence: Valence relates to voters’ judgments about positively or negatively evaluated conditions which they associate with particular parties or candidates. These judgments could refer to party leaders’ competence, integrity, moral stance or “charisma” over issues such as the ability to deal with the economy and politics. The important point to note is that these individual judgments are independent of the positions of the voter and party. Schofield and Sened (2006) review the evidence for a number of countries, and they conclude that there is no indication of convergence of the positions of party leaders.\footnote{These countries included Britain, Germany, the Netherlands, Israel, Italy and the United States.} Schofield and Cataife (2007) studied the 1989 and 1995 elections in Argentina, and also found that party platforms did not converge.

In this paper we develop a stochastic model of electoral competition to study the economic and political determinants of trade policy. We model a small open economy with two tradable goods, each of which is produced using a sector specific factor (land and capital) and a third factor (e.g., labor) which is mobile between these tradable sectors. There is also one non-tradable good, which is produced using a specific factor (skilled labor). The political model has an elected government with the mandate to fix an ad valorem import tax rate. The tax revenue is used to provide two local public goods. One public good is
targeted at the specific factors of production while the other is targeted at the mobile factor of production. We use this general equilibrium model to explicitly derive the preferences of the different socioeconomic groups in society (landlords, industrialists, workers and service workers). We then use those derived preferences for political policies to model the individual probabilistic voting behavior of the members of each of these socioeconomic groups. The combined model is thus based on micro-political economy foundations of citizens preferences. We believe this paper is the first to employ this methodology in order to study how differences in the factor endowments of countries explain trade policy divergence between countries as well as trade policy instability within countries. Trade policy instability requires that political parties diverge in equilibrium over the political economic platforms that they present to the electorate, and commit to implement if elected.

Just as in Grossman and Helpman (1994, 1996) we consider two interconnected sources of political influence: electoral competition and interest groups. In their study of the political economy of protection Grossman and Helpman proposed a model of protection in which economic interests organize along sectoral lines, so that interest groups form to represent industries. Their model predicts a cross-sectional structure of protection, depending on political and economic characteristics, and provides an excellent model of within country cross-section variability of trade policy. In contrast, we focus on the variability of trade policy both across countries and within a country over time, rather than across sectors.

Our work is related to Roemer (2001), which presents several models of political competition in which the central economic dimension is the distributive conflict among different socioeconomic groups. Acemoglu and Robinson (2005) offer a theory of political transition that uses the distributive conflict between the rich and the poor as the main driving force behind political change, and they also stress structural differences between rural elites (landlords) and urban elites (industrialists) in highlighting important equilibrium institutional differences across countries (see also Acemoglu et al. [2008]). Since we emphasize redistributive conflict as the main determinant of trade policy, our work is also related to the analysis of Rogowski (1987, 1989) on the effects of international trade on political alignments (see also Baldwin [1989]). Rogowski combines the Stolper-Samuelson theorem (Stolper and Samuelson [1941]) and Becker’s theory of competition among pressure groups (Becker [1983]) to elaborate a lucid explanation of political cleavages, as well as changes in those cleavages over time as a consequence of exogenous shocks in the risk and cost of foreign trade. The beauty of Stolper-Samuelson is that it identifies winners and losers in free trade in simple economies. For example, it explains why in nineteenth Century England capitalists and workers united in favor of free trade against the landowner elite; while American capitalists and workers did the same but with a different objective: protectionism.

Rogowski (1987) classifies economies according to their factor endowments of capital, land and labor, and uses his classification to deduce two main types of political cleavages: a class cleavage and a urban-rural cleavage. The model that we present includes non-tradable goods and this allows for a richer characterization of political alignments. In particular, in natural resource (land) abundant economies, without the inclusion of non-tradable goods, landlords favor free trade, and industrialists and workers are protectionist, inducing a urban-rural cleavage. However, once non-tradable goods are introduced in the

7 Albornoz Galiani and Heymann (2008) introduce foreign direct investment in infrastructure such as railways in the standard two sector model of a small open economy and study how the redistributive effect of the railway (triggered by Stolper-Samuelson effects) differentiates the interests of landlords and workers with respect to policies such as expropriation. Dal Bo and Dal Bo (2009) introduce appropriation activities in the two sector model of a small open economy, and instead employs the Stolper-Samuelson theorem to study how economic and policy shocks affect the intensity of appropriation activities.
model, distributive conflict among urban groups will also be present. Industrialists and unskilled workers may favor protectionist policies while skilled workers favor free trade policies (see Galiani, Heymann, and Magud [2009]). Furthermore, we show that the presence of a distributive conflict between urban groups can have interesting political effects in the determination of trade policy.

We construct a taxonomy to classify different economies given their economic structures:

1) **Natural resource-rich economies.** This set comprises countries that are highly abundant in the factor specific to the less labor-intensive tradable industry (land). They specialize in the production of primary goods.

2) **Diversified natural resource-rich economies.** They comprise countries that are moderately abundant in the factor specific to the less labor intensive tradable industry (land), but they display an important activity in the production of the two tradable goods.

3) **Industrial economies.** They comprise countries relatively scarcely endowed with natural resources that are either relatively abundant in the factor specific to the more labor-intensive tradeable industry (capital) or are highly endowed with the mobile factor of production (labor).

We show that in a natural resource abundant economy with very little capital, or in an economy with comparative advantage in the manufacturing sector (i.e., industrial economies), political parties tend to converge to the same policy platform. Trade policy is likely to be stable and relatively close to free trade. In contrast, in a natural resource abundant economy with an important domestic industry which competes with imports, parties tend to diverge. Trade policy is likely to be more protectionist and unstable. This is consistent with the empirical evidence in O’Rourke and Taylor (2006) who show that, in the late nineteenth century, democratization led to more liberal trade policies in countries where workers stood to gain from free trade. Using more recent evidence, Mayda and Rodrik (2005) show that individuals in sectors with a revealed comparative disadvantage tend to be more protectionist than individuals in sectors with a revealed comparative advantage. They also show that individuals in non-tradable sectors tend to be the most pro-trade of all workers.

We also show that when policy platforms diverge the economic structure influences the pattern of divergence. In particular, in specialized natural resource-rich and industrial economies, parties tend to propose very similar trade policies, but they differ in their budget allocation proposal. Thus, distributional conflict mainly occurs in the budget allocation, which does not affect the efficiency of the economy. On the other hand, in diversified natural resource-rich economies parties tend to differ in both dimensions. Thus, party rotation induces significant changes in the efficiency of the economy since each party implements a very different trade policy.

The rest of the paper is organized as follows. Section 2 presents our simple general equilibrium model of a small open economy. We find and characterize the competitive equilibrium of the model, as well as the induced preferences over policies of each group of agents. In section 3 we introduce the stochastic spatial electoral model with exogenous valence, and we use it to study the political economy of trade policy. Section 3.1 presents the conditions for convergence to a weighted political mean. In section 3.2 we emphasize that political convergence depends both on political parameters, such as heterogeneity of political perceptions, and on economic structure, namely the electoral covariance matrix of economic preferences. In section 3.3 we show how the structure of the economy affects policy choices, in particular the equilibrium trade policy. Section 4 extends the stochastic electoral model to include interest groups. In section 4.1 we extend the model to incorporate interest groups. In section 4.2 we study how interest groups affect the political economic equilibrium. In section 4.3 we briefly discuss how interest groups
affect political convergence. In section 4.4 we characterize an equilibrium in which parties’ platforms diverge. Finally, section 5 offers brief concluding remarks.

2 The economy

In this section we develop a static model of a small open economy and characterize the indirect utility functions of the different groups in society. Consider a factor specific static model of an open economy with two tradable goods, labeled X and Y, and a non-tradable good, labeled N. Good X (Y) is produced employing a factor specific to industry X (Y), denoted $F_X$ ($F_Y$), and labor, which can move between tradable industries without friction, denoted $L$. Let $L_X$ ($L_Y$) be the amount of $L$ employed in industry X (Y). Production functions are assumed to be Cobb Douglas with different factor intensities:

$$Q_X = A_X (F_X)^{\alpha_X} (L_X)^{1-\alpha_X},$$
$$Q_Y = A_Y (F_Y)^{\alpha_Y} (L_Y)^{1-\alpha_Y}.$$ 

We assume, without lose of generality, that $\alpha_X > \alpha_Y$. The nontradable good is produced employing labor specific to industry N, denoted $F_N$, with the linear production function:

$$Q_N = A_N F_N.$$ 

Here $Q_k$ ($k = X, Y, N$) is the total output of good k. The aggregate vector endowment of factors is $e = (F_X, F_Y, F_N, L)$.

We focus on the functional distribution of income. Therefore, we only consider four socioeconomic groups associated with the resources they control: for example, natural resources, capital, labor and skilled labor. The society we have in mind is one composed of landlords, industrialists (owning sector specific capital), workers (mobile factor between tradable industries) and service workers. We identify the later with skilled workers. A household of type $k$ owns $\frac{1}{n_k}$ units of factor $k$, and zero units of all the other factors, where $n_k$ represents the population belonging to group $k$. All individuals have the same utility function. This is Cobb Douglas in private goods and separable in a local public good:

$$u^{i,k} = (c^{i,k}_X)^{\beta_X} (c^{i,k}_Y)^{\beta_Y} (c^{i,k}_N)^{\beta_N} + H(G_k).$$

Here $c^{i,k}_l$ is the consumption of the private good $l = X, Y, N$ by individual $i$ of type $k$ ($0 < \beta_l < 1$, with $\sum_{l=X,Y,N} \beta_l = 1$). $G_k$ is the consumption of a local public good, and $H$ is an strictly increasing and concave function. Let $C^p_l$ be the aggregate consumption of private good $l = X, Y, N$ consumed by the private sector.

8It is not difficult to extend the model to any finite number of tradable goods, each produced with a specific factor and factor $L$. However, the political equilibrium would be more complicated and the fundamental message of our analysis would remain the same.

9This is clearly a simplification. The service sector tends to comprise both unskilled workers, such as domestic workers, and highly skilled workers, such as financial sector workers, medical doctors, etc. Thus, for the sake of simplicity, we are abstracting from modeling the unskilled segment of the service sector. Nevertheless, including this subsector in the model would not change the qualitative results of our analysis.
In order to avoid distorting the private good markets merely due to the public sector utilization of private goods as its inputs of production we assume that the government also has a Cobb Douglas production function with the same coefficients of the utility function

\[ Q_G = A_G \left( C_X^{\beta_X} C_Y^{\beta_Y} C_N^{\beta_N} \right). \]

Here \( C_l^q \) is the amount of good \( l = X, Y, N \) used as input by the public sector\(^{10}\), and

\[ A_G = \left[ (\beta_X)^{\beta_X} (\beta_Y)^{\beta_Y} (\beta_N)^{\beta_N} \right]^{-1}. \]

Even though we do not need this assumption to obtain our results, it simplifies the analysis below. These local public goods are just a convenient way of handling transfers in kind to different groups in society. In particular, we assume that the government provides two local public goods: one that benefits specific factors, denoted \( G_F \), and the other that benefits the mobile factor, denoted \( G_L \). These are associated, respectively, with the upper and middle-class groups and the low-income group.

Finally, we assume that the economy is small in the sense that it cannot affect the international prices of tradable goods \( p^* = (p_X^*, p_Y^*) \). Then, a feasible allocation for this economy is given as follows:

**Definition 1** A feasible allocation for this small open economy is a vector \( x = \left( (F_l, L_l, Q_l)_{l=X,Y}; (F_N, Q_N); (C_l^q)_{l=X,Y,N}; G_F, G_L \right) \) such that:

1. \( F_l \leq \bar{F}_l \) for \( l = X, Y, N \), \( L_X + L_Y \leq \bar{L}, Q_l \leq A_l (F_l)^{\alpha_l} (L_l)^{1-\alpha_l} \) for \( l = X, Y \), \( Q_N \leq A_N F_N \), \( Q_G \leq A_G \left( C_X^{\beta_X} C_Y^{\beta_Y} C_N^{\beta_N} \right) \); and
2. \( G_F + G_L \leq Q_G \), \( C_N \leq Q_N \), \( p_X^* Q_X + p_Y^* Q_Y \leq p_X^* \left( C_X^q + C_X^q \right) + p_Y^* \left( C_Y^q + C_Y^q \right) \).

### 2.1 Competitive equilibrium

Since the government can tax exports and impose import tariffs, domestic prices may differ from international prices. Let \( p = (p_X, p_Y, p_N) \) be the vector of domestic good prices, and \( w = (w_{F_X}, w_{F_Y}, w_{F_N}, w_L) \) be the vector of factor prices, where \( w_k \) is the rental rate of factor \( k \). Due to Lerner’s theorem export taxes are equivalent to import tariffs. Thus, without lose of generality, we assume that the government only taxes exports at the rate \( \tau \geq 0 \).

**Definition 2** A competitive equilibrium for an economy with endowment \( e \), international prices \( p^* \) and export tax rate \( \tau \geq 0 \) is a list \( (x, p, w) \), such that \( x \) is a feasible allocation, each firm maximizes profits given \( (p, w) \), each individual maximizes utility given \( (p, w) \), and \( p \) satisfies

\[
\begin{align*}
  p_X &= (1 - \delta_X \tau) p_X^*, \\
  p_Y &= (1 - \delta_Y \tau) p_Y^*.
\end{align*}
\]

Here \( \delta_i \) is an indicator variable that equals 1 if the economy has a comparative advantage in industry \( i = X, Y \) and 0 otherwise.

\(^{10}\) As we show in the next section, this specification does not imply that the presence of the public sector does not change the competitive equilibrium of the economy, neither that it does not affect welfare. It merely implies that the public sector, as it is our desire, only affects the economy through tax collection and the assignment of the local public goods.
Let \( \bar{Q}_X (\bar{Q}_Y) \) be the maximum output of industry \( X (Y) \) given the aggregate endowment \( e \), so \( \bar{Q}_X = A_X \left( \bar{F}_X \right)^{\alpha_X} \left( \bar{L} \right)^{1-\alpha_X}, \bar{Q}_Y = A_Y \left( \bar{F}_Y \right)^{\alpha_Y} \left( \bar{L} \right)^{1-\alpha_Y}. \) Let \( l_Y \) be the fraction of factor \( L \) employed in industry \( Y \), so \( l_Y = \frac{L_Y}{L}. \) Then, profit maximization in industry \( X \) implies

\[
p_X \alpha_X \bar{Q}_X (1-l_Y)^{1-\alpha_X} = w_{F_X} \bar{F}_X, \\
p_X (1-\alpha_X) \bar{Q}_X = w_L (1-l_Y)^{\alpha_X} L.
\]

Profit maximization in industry \( Y \) implies

\[
p_Y \alpha_Y \bar{Q}_Y (l_Y)^{1-\alpha_Y} = w_{F_Y} \bar{F}_Y, \\
p_Y (1-\alpha_Y) \bar{Q}_Y = w_L (l_Y)^{\alpha_Y} L.
\]

Finally, the zero profit condition in industry \( N \), implies

\[
A_N p_N = w_{F_N}.
\]

Under the Cobb Douglas utility assumption, expenditure shares are constant, so

\[
\frac{p_X c_X}{\beta_X} = \frac{p_Y c_Y}{\beta_Y} = \frac{p_N c_N}{\beta_N}.
\]

From profit maximization in industries \( X \) and \( Y \), and the connection between domestic and international prices we obtain

\[
(1-l_Y)^{\alpha_X} (1-\alpha_Y) (1-\delta_Y \tau) p_Y^* \bar{Q}_Y = (1-\alpha_X) (1-\delta_X \tau) p_X^* \bar{Q}_X (l_Y)^{\alpha_Y}.
\]

(1)

From profit maximization in the nontradable industry and the relation between expenditure shares we obtain

\[
p_N = \left( \frac{\beta_N}{\sum_{l=x,y} \beta_{l(1-\delta_l \tau)}} \right) \left[ p_X^* \bar{Q}_X (1-l_Y)^{1-\alpha_X} + p_Y^* \bar{Q}_Y (l_Y)^{1-\alpha_Y} \right] \frac{\beta_N}{Q_N}.
\]

(2)

Note that the right hand side of equation (1) is a positive constant while the left hand side is an strictly decreasing function of \( l_Y \). Therefore, there exists a unique \( l_Y \) that solves the equation. Once \( l_Y \) is determined, equation (2) determines a unique \( p_N \). Hence, given a vector of international prices \( p^* \), a vector of factor endowments \( e \), and a tax rate \( \tau \), equations (1) and (2) determine a unique equilibria, \( l_Y \) and \( p_N \). Denote by \( l_Y (\tau) \) and \( p_N (\tau) \) the functions that give the equilibrium values of \( l_Y \) and \( p_N \) for each \( \tau \), given \( p^* \) and \( e \). A direct application of the implicit function theorem implies that these functions are continuously differentiable. Analogously, let \( w_k (\tau) \) denotes the equilibrium nominal factor price for factor \( k \), and define the equilibrium consumer price index as the following geometric average of the prices of consumption goods

\[
CPI (\tau) = \left[ (1-\delta_X \tau) p_X^* \right]^{\beta_X} \left[ (1-\delta_Y \tau) p_Y^* \right]^{\beta_Y} \left[ p_N (\tau) \right]^{\beta_N}.
\]

(3)

**Definition 3** Define the economy degree of comparative advantage in industry \( Y \) by \( \Psi = \frac{A_Y (\bar{F}_Y)^{\alpha_Y} (\bar{L})^{(\alpha_X-\alpha_Y)}}{A_X (\bar{F}_X)^{\alpha_X}}, \) and the coefficient \( \Omega = \frac{(\beta_Y)^{\alpha_Y} (1-\alpha_X)^{1-\alpha_X}}{(\beta_X)^{\alpha_X} (1-\alpha_Y)^{1-\alpha_Y}} \left[ \beta_Y (1-\alpha_Y) + \beta_X (1-\alpha_X) \right]^{\alpha_X-\alpha_Y}.\)
Lemma 1 The economy has a comparative advantage in industry $X$ (respectively $Y$) if and only if $\Psi < \Omega \frac{p_X}{p_Y}$ (respectively $\Psi > \Omega \frac{p_X}{p_Y}$). **Proof:** See the appendix.

Let $\eta_{A,B}$ denote the elasticity of variable $A$ with respect to variable $B$. Then differentiating equations (1) and (2) we obtain:

\[
\eta_{l,\tau} = \eta_{l,p_X}\eta_{p_X,\tau} = \left[ \frac{l}{\alpha_Y (1 - l_Y) + \alpha_X l_Y} \right] \left( -\frac{\tau}{1 - \tau} \right), \quad (4)
\]

\[
\eta_{p_N,\tau} = \eta_{p_N,p_X}\eta_{p_X,\tau} = \left[ \frac{\beta_l}{\beta_l + (1 - \tau) \beta_{l-1}} \right] + (1 - \alpha_l) \theta_l \eta_{l,p_X} \left( -\frac{\tau}{1 - \tau} \right), \quad (5)
\]

where $\theta_l = \frac{p^l_X Q_l}{p_X Q_X + p_Y Q_Y}$ ($l = X, Y$) is the share of the export good in the total tradable output evaluated at international prices. The interpretation of (4) and (5) is straightforward. If the economy has a comparative advantage in industry $X$, an increase in the export tax rate decreases the domestic price of good $X$, and hence some workers leave industry $X$ and move to industry $Y$ (i.e. $l_Y(\tau)$ is a strictly increasing function). On the other hand, if the economy has a comparative advantage in industry $Y$, an increase in the export tax rate reduces the domestic price of good $Y$, and hence some workers reallocate to industry $X$. That is, $l_Y(\tau)$ is a strictly decreasing function.

No matter what the comparative advantage of the economy, an increase in the export tax rate always generates a reduction in the aggregate output of the tradable industries, measured at international prices. Since the total demand of the nontradable good is proportional to $p_X Q_X + p_Y Q_Y$, this reduction induces a contraction in the demand of the nontradable good, and hence a decrease in the price of the nontradable good. Thus $p_N(\tau)$ is a strictly decreasing function.

**Definition 4** Let $\tau_{aut}$ be the tax rate on exports that pushes the economy into autarky.\(^{11}\)

Lemma 2 **Specific Factor prices:** The real rental factor prices of the factor specific to the exporting industry and the nontradable industry are decreasing in the export tax rate for all $\tau \in [0, \tau_{aut}]$; while the real rental factor price of the factor specific to the import competing industry is increasing in the export tax rate for all $\tau \in [0, \tau_{aut}]$. **Proof:** See the appendix.

The interpretation of lemma 2 is as follows. On the one hand, an increase in the export tax rate reduces the domestic relative price of the exporting industry and hence the real rental price of the factor specific to this industry. On the other hand, an increase in the export tax rate increases the domestic relative price of the industry that competes with imports, and hence it increases the real rental price of the factor specific to that industry. The real wage paid in the nontradable industry also decreases when the export tax rate increases since the demand of the nontradable good is proportional to the income generated in the tradable industries, which varies inversely with the export tax rate.

It is more subtle to see what happen with the real wage paid in the tradable industries. There are three effects operating on the real wage, and they can operate in opposite directions.

First, the increase in the export tax rate reduces the domestic price of the exported good, and, for a given allocation of labor between tradable industries, this reduces the nominal wage in the same

\[^{11}\tau_{aut}\] depends on the factor endowments and the international terms of trade. See the appendix.
amount. Second, the export tax also reduces the domestic relative price of the exported good relative to imported good and, hence labor reallocates to the industry that competes with the imports, and this can counterbalance or reinforce the reduction in the nominal wage since industries differ in their labor intensity. Third, as the export tax rate increases there is a reduction in the domestic price of exported goods. This increases the real wage. There is also a reduction in the aggregate output of the tradable industries measured at international prices, and this reduces the price of nontradable goods, which increases the real wage.

The following lemma formally characterizes the effect of the export tax rate on real wages.

**Lemma 3 Mobile Factor prices:** Suppose that the economy has a comparative advantage in the less labor intensive industry $X$, that is $\Psi < \Omega \frac{p_X}{p_Y}$. Then, if $\Psi = 0$ the real wage is decreasing in the export tax rate for all $\tau \geq 0$, while if $\Psi > \left( \frac{\alpha_Y}{1-\alpha_Y} \right)^{\alpha_Y} \left( \frac{1-\alpha_X}{\alpha_X} \right)^{\alpha_X} \Omega \left( \frac{p_X}{p_Y} \right)$ the real wage is increasing in the export tax rate for all $\tau \in [0, \tau_{aut}]$. On the other hand, suppose that the economy has a comparative advantage in the more labor intensive industry $Y$, that is $\Psi > \Omega \frac{p_X}{p_Y}$. Then, if the following two conditions hold, the real wage is decreasing in the export tax rate for all $\tau \in [0, \tau_{aut}]$:

1. $\alpha_X \geq \max \left\{ \frac{\beta_X (1-2\beta_Y)}{\beta_X (1-2\beta_Y) + \beta_Y (1-\alpha_Y)} \cdot \frac{(\beta_Y + \beta_N) \alpha_Y}{(\beta_Y + \beta_N) \alpha_Y} \right\}$

2. $\Omega \frac{p_X}{p_Y} < \Psi \leq \frac{1}{1-\tau_{aut}} \Omega \frac{p_X}{p_Y}$, where $\tau_{aut} = \left( 1 + \frac{1}{2} \frac{\beta_Y}{\beta_X} \right) - \sqrt{\left( 1 + \frac{1}{2} \frac{\beta_Y}{\beta_X} \right)^2 - 1}$. **Proof:** See the appendix.

The interpretation of lemma 3 is as follows. If the economy is completely specialized in the production of the less labor intensive tradable good ($X$), then the nominal wage changes one to one with the domestic price of $X$, and the price of the nontradable good reduces proportionally less than the domestic price of $X$. The reason for this is that the export tax does not generate any productive distortion in the production of good $X$, and the consumption distortion has a relatively mild effect on the price of the nontradable good.

If the economy has a comparative advantage in the less labor-intensive industry, but it is not completely specialized, an increase in the export tax rate induces a reallocation of labor toward the more labor intensive industry ($Y$), which counterbalances the initial reduction in the nominal wage. For this counterbalancing effect to be of importance, the amount of the industry $Y$ specific factor must be high enough. The reduction in the price of nontradable goods associated with an increase in the export tax rate reinforces the increase in the real wage.

Finally, if the economy has a comparative advantage in the more labor-intensive industry, the export tax produces a reallocation of labor to the less labor-intensive industry, which reinforces the initial reduction in the nominal wage. Since, the price of the nontradable good also decreases, the effect on the real wage is in principle ambiguous. However, if industry $X$ is intensive enough in the specific factor (condition 1 in the lemma), and the comparative advantage in industry $Y$ is not extremely high (condition 2 in the lemma), the decrease in the nominal wage is relatively high, and the decrease in the price of the nontradable good, which is proportional to the distortion, is relatively moderate.

In principle, Lemma 3 does not exhaust all situations. For the domain $0 < \Psi < \left( \frac{\alpha_Y}{1-\alpha_Y} \right)^{\alpha_Y} \left( \frac{1-\alpha_X}{\alpha_X} \right)^{\alpha_X} \Omega \left( \frac{p_X}{p_Y} \right)$ the effect of the tax rate on the real wage is ambiguous. However, we
simulated the model for reasonable values of the parameters and we found that the real wage always decreases for low values of the tax rate and then increases for high values of the tax rate. Moreover, for strictly positive, but small values of $\Psi$, the maximum real wage is at $\tau = 0$. For higher values of $\Psi$ the maximum real wage is at $\tau = \tau_{\text{out}}$. Regarding the domain in which neither condition 1 nor condition 2 holds, we also simulated the model for reasonable values of the parameters and found that the sufficient conditions in Lemma 3 are far from being necessary. Thus, Lemma 3 and the simulations we conducted suggest the following taxonomy of economic structures:

1) **Natural resource-rich economies.** This set comprises countries that are highly abundant in the factor specific to the less labor-intensive tradable industry (land). They specialize in the production of primary goods.

2) **Diversified natural resource-rich economies.** They comprise countries that are moderately abundant in the factor specific to the less labor intensive tradable industry (land), but they display an important activity in the production of the two tradable goods.

3) **Industrial economies.** They comprise countries relatively scarcely endowed with natural resources that are either relatively abundant in the factor specific to the more labor-intensive tradeable industry (capital) or are highly endowed with the mobile factor of production (labor).

Note, however, that this taxonomy is a static one. An economy with a given endowment vector $e$ could be classified, for example, either under the category 1 or 2 depending on, among other things, the international relative price of the tradable goods (see Galiani and Somaini [2009]). Additionally, the vector endowment $e$ could evolve over time. In particular, physical and human capital accumulation have the potential to change significantly.

Many economies can be accommodated within this taxonomy. Economies highly endowed with natural resources (relative to capital and labor), such as, for example, Argentina before the 1930 crisis, or most OPEC countries, can be regarded as having a type 1 economic structure. However, Argentina, after the War World II, is better classified as having a type 2 economic structure (see Galiani and Somaini [2009]). Actually, most economies well endowed with natural resources and which adopted import substitution policies moved from a type 1 to a type 2 economic structure. Many backward economies, such as those of Africa, can also be seen to have a type 2 economic structure, even though they might not have an important industrial sector. In this case, the agricultural sector acts as the sector intensive in the use of labor ($L$), while the exporting sector exploits the endowment of a specific natural resource (e.g., diamonds in Botswana). Finally, type 3 economies consist of two types. First are those that are highly endowed with capital (relative to natural resources and labor) such as all highly developed countries. Second are those highly endowed with labor ($L$) that export labor intensive manufactured goods such as it is the case of China today.\(^\text{12}\)

### 2.2 The policy space and indirect utility functions

Real government revenue is given by

\[
\frac{R(\tau)}{CPI(\tau)} = \frac{\delta_{I} \tau p_{L}^{*} [Q_{I}(\tau) - C_{I}(\tau)]}{CPI(\tau)},
\]

\(^{12}\text{Note that in the case of developed economies highly abundant in capital, all our results will still hold even if it were the case that the workers in the tradable exporting sector are skilled and can move without friction between this industry and the (skilled) service sector.}\)
where $Q_l(\tau)$ and $C_l(\tau)$ measure respectively the equilibrium production and consumption of good $l = X, Y$. $\frac{R(\tau)}{CPI(\tau)}$ has the typical inverted U shape with zeros at $\tau = 0$ and $\tau = \tau_{aut}$ and a maximum at $\tau_{max}$ given by

$$\frac{1 - \tau_{max}}{\tau_{max}} = \delta_l \left[ \eta_l(Q_l) - C_l \right] \eta_{nl} p_{nl} - \beta_l \right].$$

(7)

In equilibrium, $Q_G = \frac{R(\tau)}{CPI(\tau)}$. Suppose, however, that a fraction of the public goods vanishes in the process of distributing it, possibly due to corruption or any other form of rent dissipation prevalent in the operation of the public sector. This assumption, though realistic, is not crucial in deriving the results of the paper below but it facilitates the analysis. Then,

$$G_L = A(\gamma) \frac{R(\tau)}{CPI(\tau)}, G_F = A(1 - \gamma) \frac{R(\tau)}{CPI(\tau)},$$

(8)

where $\gamma \in [0, 1]$ is the fraction of government revenue allocated to the provision of $G_L$ ($1 - \gamma$ is the fraction allocated to $G_F$), and $A(.)$ is an strictly increasing and concave function such that: $A(\gamma) \leq \gamma$, $A(0) = 0$, and $A'(1) = 0$.

From equations (6) and (8) we see that public decisions are restricted to a two dimensional space: the government must set the export tax rate and the fraction of revenue assigned to the provision of each local public good.

**Definition 5** The **policy space** of an economy with endowment vector $e$ and international prices $p^*$ is given by

$$Z = \{ z = (\tau, \gamma) : 0 \leq \tau \leq \tau_{aut}, 0 \leq \gamma \leq 1 \} \subset \mathbb{R}_+^2.$$

(9)

Here $\tau$ is the tax rate on exports and $\gamma$ is the fraction of government revenue allocated to the provision of $G_L$.

Clearly, $Z$ is a convex and compact subset of the semipositive quadrant $\mathbb{R}_+^2$. Since preferences are represented by Cobb Douglas utility functions, the indirect utility function of each individual is given by a geometric average of his real income using the consumer price index as deflator and the amount of the local public good consumed by the individual. Formally,

**Definition 6** The **indirect utility function** of an individual belonging to group $k = (F_X, F_Y, F_N, L)$ is given by

$$v^k(\tau, \gamma) = \frac{w_k(\tau)}{CPI(\tau)} \frac{\bar{k}}{n_k} + H \left( A(\gamma_k) \frac{R(\tau)}{CPI(\tau)} \right).$$

(10)

where $\gamma_L = \gamma$, and $\gamma_k = 1 - \gamma$ for $k = F_X, F_Y, F_N$.

For each group in society, its ideal policy is the point in the policy space $Z$ that maximizes its indirect utility function (10).

\[\text{Note that } \tau_{max} \text{ depends on the factor endowments and the international terms of trade.}\]
Lemma 4 Ideal policies. Let \( z^k = (\tau^k, \gamma^k) \) denote the ideal policy for an individual from group \( k \). Then \( \gamma^k = 0 \) for \( k = F_X, F_Y, F_N \), \( \gamma^L = 1 \). Moreover, assume that \( \lim_{G \to 0} H'(G) = -\infty \) and \( \max \{ \frac{w_{F_X}}{n_{F_X}}, \frac{w_{F_Y}}{n_{F_Y}} \} \geq \frac{w_{L}}{n_{L}} \). Then, for economies characterized by structure 1, \( \tau^{F_X} < \tau^{F_N} < \tau^L < \tau^Y \). For economies characterized by structure 2, \( \tau^{F_X} < \tau^{F_N} < \tau^{L} < \tau^Y \). For economies characterized by structure 3, \( \tau^{F_Y} < \tau^{F_N} < \tau^L < \tau^{\max} < \tau^{F_X} \). Proof: see the appendix.

The ideal policy for each socioeconomic group is the key economic input of the political game that we develop in the next section. Note, in particular, how these ideal policies vary with different economic structures. In an economy highly abundant in the factor specific to the less labor intensive tradable industry (structure 1), the only protectionist demand appears from the group that owns the factor specific to the import competing industry. For an economy abundant in the factor specific to the more labor-intensive tradable industry (structure 3), the only protectionist group is the one that owns the factor specific to the import competing industry. However, in an economy moderately abundant in the factor specific to the less labor intensive tradable industry (structure 2), there are a number of possible opposed groups. There are two protectionist groups, those owning \( F_Y \) or \( L \), while the groups owning \( F_X \) or \( F_N \) lose from protection.

3 The Polity

In this section we introduce the stochastic spatial model of electoral competition. We begin with a formal definition of the stochastic spatial model as a game in normal form. We define and discuss an equilibrium concept for this game, and study the conditions under which parties converge to a weighted electoral mean. We then use the model to study the political determination of trade policies in the economy we study.

3.1 The stochastic spatial model with exogenous valence

The timing of events is as follows (Person and Tabellini [2000]):

1. Party leaders simultaneously announce their electoral platforms.
2. Each voter receives a private signal about candidates’ valence.
3. Elections are held.
4. The elected candidate implements the announced platform.

Let \( P = \{1, \ldots, p\} \) be the set of all political parties. Each party \( j \in P \) selects a platform \( z_j = (\tau_j, \gamma_j) \) from the policy space \( Z \). We let \( Z = \times_{j \in P} Z \). A profile of party platforms is denoted \( z \in Z \). When necessary we use the notation \( z_{-j} \) to represent the profile of platforms of all parties except party \( j \). The preferences of party \( j \in P \) is given by its expected vote share function \( S_j : Z \to [0, 1] : \)

\[
S_j(z) = \sum_{k \in V} n_k \rho^k_j(z).
\]

(11)

Here \( \rho^k_j(z) \) is the probability that a voter in group \( k \) votes for party \( j \), while \( V = \{F_X, F_Y, F_N, L\} \) is the set of all groups of voters, and \( n_k \) is the proportion of the population in group \( k \).
The utility associated with a given voter in group $k$ when party $j$ implements platform $z_j$ is given by

$$v^k(z_j) = v^k_{pol}(z_j) + \lambda_j + \varepsilon^k_j$$

$$= -\phi^k_\tau \left( \tau_j - \tau^k \right)^2 - \phi^k_{\gamma_j} \left( \gamma_j - \gamma^k \right)^2 + \lambda_j + \varepsilon^k_j,$$  \hspace{1cm} (12)

where (a) $z^k = (\tau^k, \gamma^k) \in Z$ is the ideal policy for the voters in group $k$; (b) $\phi^k_\tau > 0$ ($\phi^k_{\gamma_j} > 0$) measures the importance that voters in group $k$ assign to the export tax rate (the local public good); and (c) $\lambda_j + \varepsilon^k_j$ is the private signal received by a voter in group $k$ about party $j'$s valence. We shall assume that the expected value of this signal is $\lambda_j$, and is common to all groups$^{14}$, and the error vector $\varepsilon^k = (\varepsilon^k_1, ..., \varepsilon^k_p)$ has a cumulative stochastic distribution denoted $F^k$. We assume below that $F^k$ is the Type 1 extreme value distribution, the same for all $k$.

The utility function (12) requires some explanation. Note that we do not use the indirect utility functions over policies (10) to capture the preferences of the different socioeconomic groups. Instead we use a weighted Euclidean metric, given by the distance from the policy proposed by the $j^{th}$ party, $z_j$, to the optimal policy, $z^k$, for each group $k$. The model can be developed using the indirect utility functions, at the cost of analytical tractability (since the true indirect utility functions have very complicated expressions, or even no closed form solution). Additionally, the convergence theorems that we use below have a much easier implementation under the weighted Euclidean preference assumption adopted here. Furthermore, it is possible to justify the weighted Euclidean preferences (12) as a second order Taylor approximation of the indirect utility function of each group (10) around their preferred policies:

$$v^k(z_j) - v^k(z^k) \approx Dv^k(z^k) \left( z_j - z^k \right) + \frac{1}{2} \left( z_j - z^k \right)' D^2v^k(z^k) \left( z_j - z^k \right).$$

Here the operator $D$ ($D^2$) indicates derivative of order 1 (2), and $Dv^k(z^k) = 0$ since $z^k$ is the ideal policy for voters in group $k$. Our Euclidean metric approximation assumes that $D^2v^k(z^k)$ is a diagonal matrix, which holds in our case since $\frac{\partial^2 v^k}{\partial z^2_j}(z^k) = 0$ for all $k$.

Given a profile of platforms $z \in Z$, let $v^k(z) = (v^k(z_1), ..., v^k(z_p))$. Candidates do not know the private signal received by each individual voter, but the probability distribution of these signals in each group of the electorate is common knowledge. Let $F^k$ be the cumulative distribution function of $(\varepsilon^1_k, ..., \varepsilon^p_k)$. Then the probability that a voter in group $k$ selects party $j$ is given by

$$\rho^k_j(z) = \Pr \left[ v^k(z_j) > v^k(z_l) \text{ for all } l \neq j \right].$$  \hspace{1cm} (13)

Here Pr is inferred from the cumulative distribution function, $F^k$. Finally, we order parties according to their expected valence: $\lambda_p \geq ... \geq \lambda_1$.

**Definition 7** The stochastic spatial model with exogenous valence is the game in normal form $\Gamma_{exo.} = (P, Z, S)$, where

1. **Players**: $P = \{1, ..., p\}$ is the set of political parties.

$^{14}$ Schofield et al. (2010a) develop a model where the different groups receive signals with different expected values.
2. **Set of strategies**: $Z$ is the policy space defined in section 2 and $Z = \times_{j \in P} Z$ is the space of all strategy profiles.

3. **Utility functions**: $S_j : Z \rightarrow [0, 1]$ is the expected vote share function of party $j \in P$ deduced from (12) and (13) and $S = \times_{j \in P} S_j$.

We solve this game by finding its local Nash equilibrium.

**Definition 8** A strict (weak) local Nash equilibrium of the stochastic spatial model $\Gamma_{exo} = \langle P, Z, S \rangle$ is a vector of party positions $z^*$ such that for each party $j \in P$, there exists an $\epsilon$-neighborhood $B_\epsilon(z_j^*) \subset Z$ of $z_j^*$ such that

$$S_j (z^*_j, z^*_{-j}) > (\geq) S_j (z'_j, z^*_{-j}) \text{ for all } z'_j \in B_\epsilon(z_j^*) - \{z_j^*\}.$$

**Remark 1.** A local Nash equilibrium is a pure strategy Nash equilibrium (PNE) if we can substitute $Z$ for $B_\epsilon(z_j^*)$ in the above definition.

**Remark 2.** It is usual in general equilibrium theory to use first order conditions, based on calculus techniques, to determine the nature of the critical equilibrium. Because production sets and consumer preferred sets are usually assumed to be convex, the Brower’s fixed point theorem can then be used to assert that the critical equilibrium is a Walrasian equilibrium. However, in political models, the critical equilibrium may be characterized by positive eigenvalues for the Hessian of one of the political parties. As a consequence the utility function (expected vote share function) of such a party will fail pseudo-concavity. Therefore, none of the usual fixed point arguments can be used to assert existence of a "global" Nash equilibrium. For this reason we use the concept of a "critical Nash equilibrium", namely a vector of strategies which satisfies the first order condition for a local maximum of the utility functions of the parties. A "Local Nash Equilibrium" (LNE) satisfies the first order condition, together with the second order condition that the Hessians of all parties are negative (semi-) definite at the vector that satisfies the first order condition. Clearly, this local Nash property is necessary for a vector to be a Nash equilibrium. Once the LNE are determined, then simulation can be used to determine if one of them is a Nash equilibrium.

We are interested in studying the conditions under which political parties converge to the same platform, or else diverge and offer different platforms to the electorate. Although it is possible to consider several specifications for the distribution function of the valence signals we only develop a relatively simple version that assumes that the valence signals are distributed according to the Type 1 extreme value distribution (see Schofield and Sened [2006]; Schofield [2007]). This assumption has the advantage that it is the usual stochastic assumption used in conditional logit models of elections.

Let $(\phi_\tau, \phi_\gamma) = \sum_{k \in V} n_k (\phi_\tau^k, \phi_\gamma^k)$ be the average importance that voters give to the tax rate and the local public goods, respectively. Then, define the **weighted mean of the electoral ideal policies**, or weighted electoral mean $z_m = (\tau_m, \gamma_m)$ by

$$(\tau_m, \gamma_m) = \sum_{k \in V} n_k \left( \frac{\phi_\tau^k}{\phi_\tau}, \frac{\phi_\gamma^k}{\phi_\gamma} \right).$$

Note that $z_m$ is just a weighted average of the ideal policies of each group, where the weights take into account the fraction of voters in each group ($n_k$) and the importance that each group gives to each
policy dimension relative to the average importance in the population \((\phi^k_x/\phi_x \text{ and } \phi^k_\gamma/\phi_\gamma)\). We call 
\(z_m = \times_{j \in P} z_m \in Z\) the joint weighted electoral mean of the stochastic spatial model.

Under the assumption of the Type 1 extreme value distribution, the probability that a voter in group 
k votes for party \(j\) at a profile \(z \in Z\) can be shown to be:

\[
\rho^k_j(z) = \left[1 + \sum_{l \neq j} \exp \left( v^k_{pol.}(z_l) - v^k_{pol.}(z_j) + \lambda_l - \lambda_j \right) \right]^{-1}.
\]

The objective of party \(j\) is to maximize its expected vote share, that is

\[
\max_{z_j \in Z} S_j(z) = \sum_{k \in V} n_k \rho^k_j(z).
\]

Since \(S_j(z)\) is continuously differentiable we can use calculus to solve this problem. The first order
necessary condition is

\[
DS_j(z) = -2 \sum_{k \in V} n_k \rho^k_j(z) \left( 1 - \rho^k_j(z) \right) \left( \frac{\phi^k_x (\tau_j - \tau^k)}{\phi^k_x (\gamma_j - \gamma^k)} \right) = 0, \quad (15)
\]

If all candidates adopt the same policy position, so \(z_0 = \times_{j \in P} z_0\), say, then \(\rho^k_j(z_0)\) is independent of \(k\) and may be written \(\rho_j(z_0)\). Assuming that \(\rho_j(z_0) \neq 0\), the first order condition becomes

\[
DS_j(z_0) = -2 \rho_j(z_0) \left( 1 - \rho_j(z_0) \right) \sum_{k \in V} n_k \left( \frac{\phi^k_x (\tau_j - \tau^k)}{\phi^k_x (\gamma_j - \gamma^k)} \right) = 0,
\]

Thus

\[
\left( \tau_j \sum_{k \in V} n_k \phi^k_x - \sum_{k \in V} n_k \phi^k_x \tau^k \right) \left( \gamma_j \sum_{k \in V} n_k \phi^k_\gamma - \sum_{k \in V} n_k \phi^k_\gamma \gamma^k \right) = 0,
\]

So

\[
(\tau_j, \gamma_j) = \sum_{k \in V} n_k \left( \frac{\phi^k_x}{\phi_x} \tau^k, \frac{\phi^k_\gamma}{\phi_\gamma} \gamma^k \right) = (\tau_m, \gamma_m), \text{ for all } j.
\]

Therefore, if each party proposes \(z_m = (\tau_m, \gamma_m)\), the first order condition of all parties is satisfied. We say that the joint weighted electoral mean, \(z_m\), satisfies the first order condition for LNE.

The second order sufficient (necessary) condition for equilibrium at \(z\) is that the matrix \(D^2 S_j(z)\) evaluated at \(z\) be negative definite (semidefinite). Earlier results in Schofield (2007) can be generalized to show that

\[
D^2 S_j(z) = 2 \sum_{k \in V} n_k \rho^k_j(z) \left( 1 - \rho^k_j(z) \right) \left[ 2 \left( 1 - 2 \rho^k_j(z) \right) \mathbf{W}^k \mathbf{B}^k_{zz} \mathbf{W}^k - \mathbf{W}^k \right], \quad (16)
\]

where

\[
\mathbf{W}^k = \begin{bmatrix} \phi^k_x & 0 \\ 0 & \phi^k_\gamma \end{bmatrix}, \quad \mathbf{B}^k_{zz} = \begin{bmatrix} (\tau_j - \tau^k)^2 & (\tau_j - \tau^k) (\gamma_j - \gamma^k) \\ (\tau_j - \tau^k) (\gamma_j - \gamma^k) & (\gamma_j - \gamma^k)^2 \end{bmatrix}.
\]

\(15\) If for all groups \(\phi^k_x = \phi_x \text{ and } \phi^k_\gamma = \phi_\gamma\), then \(z_m = (\tau_m, \gamma_m) = \sum_k n_k (\tau^k, \gamma^k)\) is a weighted average of the ideal points of each group of voters, where the weights are the sizes of the groups.
Definition 9  Considering the model $\Gamma_{\text{exo.}} = (P, Z, S)$ when $F^k$ is the Type 1 extreme value distribution for all $k$, we define:

1. The probability $\rho_j (z_m)$ that a voter in group $k$ votes for party $j$ at the profile $z_m$ is
   \[ \rho_j (z_m) = \left[ 1 + \sum_{l \neq j} \exp (\lambda_l - \lambda_j) \right]^{-1} \]

   (Note that $\rho_j (z_m)$ only depends on the valence terms, and not on the party platforms.)
2. Define the coefficient $A_j$ of party $j$ by
   \[ A_j = 2 (1 - 2 \rho_j (z_m)) \]
3. The matrix $\sum_{k \in V} n_k (W^k B^k_{z_m} W^k)$ is termed the weighted electoral variance-covariance matrix about the joint electoral mean, $z_m$.
4. The characteristic matrix of party $j$ at $z_m$ is
   \[ H_j (z_m) = \sum_{k \in V} n_k \left( A_j W^k B^k_{z_m} W^k - W^k \right) \]
5. Let $\Phi = \left[ \begin{matrix} \phi_x & 0 \\ 0 & \phi_y \end{matrix} \right]$.
6. Define the convergence coefficients of the model to be
   \[ c (\Gamma_{\text{exo.}}) = A_1 \sum_{k \in V} n_k \text{Tr} (\Phi^{-1} W^k B^k_{z_m} W^k), \]
   \[ d (\Gamma_{\text{exo.}}) = A_1 \sum_{k \in V} n_k \frac{\text{Tr} (W^k B^k_{z_m} W^k)}{\text{Tr} (\Phi)}. \]

   Here $\text{Tr}(M)$ means the trace of the matrix $M$.

A result of Schofield (2007) can be generalized to the case here, of multiple groups in the economy, to show that the Hessian, $D^2 S_j (z_m)$ of party $j$ at $z_m$ can be expressed in terms of the characteristic matrix. Thus

\[ D^2 S_j (z_m) = 2 \rho_j (z_m) (1 - \rho_j (z_m)) H_j (z_m) = 2 \rho_j (z_m) (1 - \rho_j (z_m)) \sum_{k \in V} n_k \left( A_j W^k B^k_{z_m} W^k - W^k \right). \]

The following proposition establishes necessary and sufficient conditions for the joint weighted electoral mean to be an equilibrium of the electoral game.

Proposition 1  Assume that $F^k$ is the Type 1 extreme value distribution for all $k$. A sufficient condition for the joint weighted electoral mean, $z_m$, to be a strict local Nash equilibrium of the stochastic spatial model $\Gamma_{\text{exo.}} = (P, Z, S)$ is $c (\Gamma_{\text{exo.}}) < 1$. A necessary condition for $z_m$ to be a local Nash equilibrium is $d (\Gamma_{\text{exo.}}) \leq 1$. Proof: See the appendix.
If the convergence conditions hold, then the equilibrium prediction of the outcome of the electoral game is the weighted electoral mean of the ideal points \( z_m = (\tau_m, \gamma_m) \). Although it does not follow directly from the Proposition 1, simulation of a number of such electoral games shows that, when the sufficient convergence condition is satisfied, then \( z_m \) is the unique PNE. There can be two or more parties and the expected vote share of each party may differ, but the policy outcome will not be affected, since all parties implement \( z_m \). Thus, different policies can only be the consequence of differences in the economic and political parameters that determine \( z_m \). On the other hand, if the necessary convergence condition fails, then parties’ platforms do not converge in equilibrium. In this case, different policies have a positive probability of being implemented.

Next, we study how the economic structure and the parameters of the electoral game affect \( z_m \) and the convergence coefficients.

### 3.2 Trade policy under convergence

We first consider the situation in which the sufficient condition for convergence holds. Then Proposition 1 implies that the outcome of the electoral game is the weighted electoral mean \( z_m = (\tau_m, \gamma_m) \). We now characterize \( z_m \) for the three economic structures identified in section 2. From lemma 3, it is always the case that \( \gamma^k = 0 \) for \( k = F_X, F_Y, F_N \) and \( \gamma^L = 1 \). Therefore, \( \gamma_m = n_L \phi^L / \phi_\gamma \), which only depends on the parameters of the electoral game and not on those parameters that define the different economic structures. Moreover, it is not difficult to see how the parameters of the electoral game affect \( \gamma_m \). *Ceteris paribus*, the higher the fraction of workers in the tradable industries in the population (\( n_L \)), and the more sensitive they are to changes in the provision of the local public good, measured by \( (\phi^L / \phi_\gamma) \), the higher the fraction of the government revenue expended in \( G_L \) in equilibrium.\(^{16}\)

Conversely, the ideal export tax rate for each group varies across the different economic structures. From lemma 3, we know that for a structure 1 economy, with highly abundant factor \( F_X \) (e.g., land), then \( \tau^F < \tau^\max \) and \( \tau^L < \tau^\max \), while for a structure 2 economy we have \( \tau^F < \tau^\max < \tau^L \). Therefore, the electoral equilibrium \( \tau_m \) would be lower in an economy with structure 1 than in one with structure 2. Moreover, it is likely that the magnitude of this difference would be large. To see this note that, in a natural resource-abundant economy, all socioeconomic groups, except for the owners of factor \( F_Y \) (e.g., industrialists), have an ideal export tax rate below \( \tau^\max \). Hence, unless the owners of factor \( F_Y \) are much more responsive to tax policy changes than the rest of the voters, \( \tau_m \) is strictly less than \( \tau^\max \). In fact it can be very low. For example, the negative impact of the export tax on real wages in the tradable industries can be large. However, in an economy with structure 2, workers in the tradable industries have an ideal tax rate above \( \tau^\max \), so it can even be the case that in equilibrium \( \tau_m > \tau^\max \). For example, the workers in the tradable industries may be an important fraction of the population as well as being highly responsive to trade policies.

An economy with structure 3 is analogous to an economy with structure 1, with the ideal export tax rates of the owners of factors \( F_X \) and \( F_Y \) reversed. Therefore, \( \tau_m \) is also lower for an economy with structure 3 than for an economy with structure 2. Finally, note that irrespective of the economic structure, *ceteris paribus*, the higher the fraction of service workers in the population (\( n_{F_N} \)), or the more sensitive they are to changes in the export tax rate, measured by \( (\phi^F_{\gamma} / \phi_\gamma) \), the lower the equilibrium tax rate.

\[^{16}\text{If } \phi^k_\gamma = \phi_\gamma \text{ for all } k, \text{ then } \gamma_m = \frac{n_L}{\Gamma^{-n_L}}.\]
This is particularly relevant for economies with structure 2. Thus, it is not the case that natural resource abundant economies will necessarily have populist political cleavages as postulated in Rogowski (1987, 1989).

In summary, if the economy is either highly abundant in the factor specific to the less labor intensive tradable industry (structure 1), or either abundant in the factor specific to the more labor intensive tradable industry or in labor \( L \) (structure 3), the electoral equilibrium is likely to be relatively close to free trade. In this case, the great majority of the population loses with the adoption of protectionist policies. However, if the economy resembles the characteristics of the economic structure 2, society is split into two groups: owners of factor \( F_X \) and service workers who favor a relatively free trade policy, while owners of factor \( F_Y \) and workers \( L \) in the tradable industries prefer a more protectionist policy. The equilibrium tax rate is higher in this third case than in the first two cases, and so is the level of distortion in the economy. The development of the non-tradable sector plays a key role in political cleavages however. The reason is that service workers push the political equilibrium toward the ideal position of the relative abundant factor in the economy. Therefore, they act as a moderating force against the protectionist tendency.

3.3 Economic structure and divergence

As we showed in the previous section, given that the convergence condition holds, we can then explain how trade policy at a given time depends on the prevalent economic structure. Now, we investigate the convergence conditions under the three different economic structures derived in Section 2 and study how different economic structures affect the stability of trade policy.

First of all, however, we need to define what we mean by stability of a policy in our model. We interpret convergence of political parties to the same political platform as stability of policies. Indeed, if in equilibrium all political parties converge to the same platform, although there can be uncertainty about which party wins the election, there is complete certainty about the policy outcome. If, instead, in equilibrium the political parties do not converge to the same platform, then there are different policies with positive probability of being implemented. This means that we could observe different policies in a given economy over time. In this sense, an economic structure that induces political convergence is one that gives rise to stable policy outcomes. These will change smoothly in response to shocks to the distribution of political power, the international terms of trade or technology. An economic structure that induces political divergence is one that generates a more volatile environment, where we can observe (possibly large) changes in policies even without any change in the economic or political fundamentals.

Proposition 1 show that a sufficient condition for convergence to \( z_m \) is \( c(\Gamma_{exo}) < 1 \), while a necessary condition is \( d(\Gamma_{exo}) \leq 1 \). These convergence coefficients, \( c(\Gamma_{exo}) \) and \( d(\Gamma_{exo}) \), depend on the stochastic distribution of the valence signals as well as the distribution of the ideal policies in the population. We now compare the convergent coefficients for different economic structures. Since the key difference among economic structures is the ideal trade policy for the workers of the tradable industries, we consider \( d(\Gamma_{exo}) \) as a function of \( \tau^L \), keeping constant all the other variables that determine it. Note that \( d(\Gamma_{exo}) \) is a quadratic and symmetric function and has a minimum at the value of \( \tau^L \) that satisfies the following equation

\[
\frac{\partial d(\Gamma_{exo})}{\partial \tau^L} = 2A_1 n_L \phi^L_{\tau} \left[ -\phi^L_{\tau}(\tau_m - \tau^L) + \frac{1}{\phi_{\tau}} \sum_{k \in V} n_k \left( \phi^k_{\tau} \right)^2 (\tau_m - \tau^k) \right] = 0.
\]
The second term in the squared brackets is very small in absolute value (in fact, it equals zero if $\phi_k^L$ is the same for all groups). Hence, $\frac{\partial d}{\partial \Gamma_{exo}}$ depends primarily on $\tau_m - \tau^L$. If the economy has structure 1, then $\tau^F_X < \tau^F_N < \tau^L < \tau_{\text{max}} < \tau^F_Y$, which implies that unless $n_{FX} \gg n_{FY}$, $(\tau_m - \tau^L)$ is positive but very small. Therefore, for an economy with structure 1, $\frac{\partial d}{\partial \Gamma_{exo}} \approx 0$ and hence $d(\Gamma_{exo})$ is very close to its minimum. This is also the case for economies with structure 3. On the other hand, for an economy with structure 2, $\tau^F_X < \tau^F_N < \tau_{\text{max}} < \tau^L < \tau^F_Y$, which implies that unless $n_{FX} << n_{FY}$, $(\tau_m - \tau^L)$ is negative and large in absolute value, and hence $d(\Gamma_{exo})$ is far from its minimum. Since $\frac{\partial c}{\partial \tau^F_X} = \frac{\partial d(\Gamma_{exo})}{\partial \tau^F_X}/\phi_x$, the same argument also apply to the coefficient $c(\Gamma_{exo})$.

Thus, convergence coefficients tend to be larger than their minimum values for diversified natural resource-rich economies (structure 2) but very close to their minimum values for natural resource-rich economies (structure 1) and industrial economies (structure 2). If the convergence coefficients for a particular polity are large, then we can say, informally, that the likelihood of convergence is lower. This allows us to infer that policy stability in economies with structures 1 or 3 is more likely than in economies with structure 2.

The above argument has focused on the dependence of the convergence coefficients on the weighted electoral variance-covariance matrix. It is also worth mentioning that the convergence coefficients also depend on the parameters of the electoral game. Under the assumptions made on the stochastic distribution, we see that $A_1 = 2 (1 - 2 \rho_1(\mathbf{z}_m))$, so if $\rho_1(\mathbf{z}_m)$ is small then $A_1$ is large, as are the convergence coefficients. Thus convergence is less likely the greater the difference in exogenous valences. In particular, in a two party system, if $\lambda_2 \approx \lambda_1$ then the model predicts policy convergence. On the other hand, in a fragmented polity, with small low valence parties, one expects policy divergence.\(^{18,19}\)

Thus, political divergence is a consequence of both political and economic forces. Policy divergence is a pure political issue related to electoral competition. Voters have different perceptions of the average quality of the political parties, and these are independent of the platform they propose. These perceptions affect voting probabilities in such a way that candidates or party leaders need not locate at the center of the policy space. However, differences in valences alone are not enough to induce political divergence. As proposition 1 clearly shows, the convergence coefficients depend on the electoral variance-covariance matrix. If the trace of this matrix is large, then convergence is less likely. Politics makes policy divergence possible, but economic forces are needed to induce it, since it is heterogeneity in policy preferences that fundamentally determines the convergence coefficients.

### 4 Extension: Parties and Organizations

In this section we extend the stochastic spatial model of electoral competition presented in Section 3 by including organizations that try to influence political outcomes through campaign contributions. We

---

\(^{17}\)For example, empirical analysis of the 2000 presidential election in the U.S. estimated $c(\Gamma_{exo})$ to be 0.37 (Schofled et al. [2010b]). This analysis estimated the valences of the candidates on the basis of a two dimension policy model.

\(^{18}\)For example, empirical analysis of the 2002 election in Turkey estimated $c(\Gamma_{exo})$ to be 5.49, while for the 1996 election in Israel it was 3.98 (Schofled et al. [2010a,b], respectively). Both polities are highly fragmented. The analyses estimated the valences of the party leaders on the basis of a two dimension policy model involving economic issues and nationalism. The policy spaces were different from those analyzed here. Nonetheless, the results are indicative of the point made here.

\(^{19}\)Similar results hold under the assumption that valence signals have a stochastic Gaussian distribution, rather than the Type I.
formally define this extension as a two stage dynamic game and define an equilibrium concept for this dynamic game. We then study the convergence conditions and characterize the equilibrium outcome of the political game when there is no convergence. There are two motivations for introducing organizations into the basic political model developed in Section 3. First, without their inclusion, when the convergence conditions do not hold, we can say little about the electoral outcome beyond divergence. Second, even in a perfect democracy, the political power of groups differs from the extent of their political power in terms of their numbers alone.

### 4.1 The stochastic spatial model with exogenous and endogenous valence

We now assume that there exist political organizations other than political parties. These organizations are independent, with their own agenda, but may be linked to parties in various ways. An example is that of unions, which try to influence political outcomes through campaign contributions. Contributions are valuable for politicians because they can be used to increase the electorate’s perceived quality of a candidate or to discredit political rivals. Thus, valence becomes an endogenous variable that depends on campaign contributions. Grossman and Helpman (1996) consider two distinct motives for interest groups: "Contributors with an electoral motive intend to promote the electoral prospects of preferred candidates. Those with an influence motive aim to influence the politicians’ policy pronouncements."

The model presented here is a generalization of Schofield (2006), which in principle, captures both motives, as suggested in Miller and Schofield (2003, 2008) and Schofield and Miller (2007). However, in the proposition presented below we consider an example that captures the electoral motive, but not the influence motive. Except for the introduction of these organizations, the stochastic spatial model remains fundamentally the same as the model with exogenous valence.

As before, the timing of the events is as follows:

1. Organizations simultaneously announce their campaign contribution functions, specifying the contributions they will make in response to the party electoral platforms.
2. Political parties simultaneously announce their electoral platforms.
3. Organizations observe these platforms and simultaneously implement their campaign contributions.
4. Each voter receives a private signal about candidates’ quality.
5. Elections are held.
6. The elected party implements the announced platform.

Suppose that each group of voters has an organization that can make contributions to political campaigns, and assume that due to institutional constraints, political parties cannot transfer money or resources to organizations, so contributions must be nonnegative. Let $c_k : Z \to \times_{j \in P} \mathbb{R}^+ = C$ denote a contribution function made by organization $k$, and let $C^*$ denote the space of all feasible contribution functions. Let $C^* = \times_{k \in V} C^*$. A profile of contribution functions is denoted by $\mathbf{c}^* = \times_{k \in V} c_k$. When necessary we use the notation $\mathbf{c}^*_{-k}$ to denote the profile of contribution functions of all organizations except $k$.

The utility of a voter belonging to group $k$ when party $j$ implements platform $z_j$ is now

$$v^k(z_j, \mathbf{c}) = v^k_{\text{pol}}(z_j) + \lambda_j + \varepsilon_j + \mu_j(\mathbf{c}),$$

$$= -\phi^k_\tau (\tau_j - \tau^k)^2 - \phi^k_\gamma (\gamma_j - \gamma^k)^2 + \lambda_j + \varepsilon_j + \mu_j(\mathbf{c}).$$

(17)
The last term is the endogenous valence function $\mu_j : \mathbb{C} \to \mathbb{R}_+$, which captures the impact of contributions on valence values.\(^{20}\)

As before, the probability that a voter from group $k$ votes for party $j$ is given by:

$$\rho_k^j (z, c) = \Pr \left[ v^k (z_j, c) > v^k (z_l, c) \text{ for all } l \neq j \right]. \quad (18)$$

We assume that each organization has a leader, who collects contributions from its members and uses them to support political parties in their electoral campaigns. Each leader receives a "payment" that depends linearly on the policy preferences of the members of the organization, and must pay the cost of collecting the contributions among its members. Following Persson and Tabellini (2000) we assume that these costs are a quadratic function in the per member contribution since the free rider problem in collective action is more severe in large groups. The leader maximizes his expected payment net of the costs of collecting contributions. Thus, the preference of leader $k$ is given by the function $L_k : \mathbb{C} \times \mathbb{C} \to \mathbb{R}$

$$L_k (z, c) = \sum_{j \in P} S_j (z, c) \left( a_{k,j} v^k_{\text{pol}} (z_j) + b_{k,j} \right) - \sum_{j \in P} \frac{1}{2} \left( \frac{c_{k,j}}{n_k} \right)^2. \quad (19)$$

Here $c_{k,j}$ denotes the contribution made by organization $k$ to party $j$. We assume that $a_{k,j} \geq 0$ and $b_{k,j} \geq 0$. This specification is flexible enough to capture very different situations. If group $k$ does not have an organization then we set $a_{k,j} = b_{k,j} = 0$ for all $j \in P$. If the leader of organization $k$ has party preferences for party $j$ then $a_{k,j} > a_{k,l}$ and/or $b_{k,j} > b_{k,l}$.\(^{21}\) If leader $k$ is twice more effective collecting contributions than leader $h$, then $a_{k,j} = 2a_{h,j}$ and $b_{k,j} = 2b_{h,j}$. For the purposes of this paper the crucial distinction is between partisan organizations and non-partisan organizations. Since each organization "represents" the interest of a socioeconomic group, if each organization is attached to a party (i.e. the leader has a strong predilection for a particular party), then the party must indirectly adopt the policy preferences of this organization as the party preferences, at least to some extent.\(^{22}\)

**Definition 10** The stochastic spatial model with exogenous and endogenous valence is the two stage dynamic game $\Gamma_{\text{end.}} = (P, V, Z, C, S, L)$, where:

1. **Players**: $P = \{1, \ldots, p\}$ is the set of all political parties, and $V = \{F_X, F_Y, F_N, L\}$ is the set of all groups of voters, which is also the set of all organization leaders.

2. **Utility functions**:

   (a) $S_j : \mathbb{Z} \times \mathbb{C} \to [0, 1]$ is the expected vote share function of party $j \in P$, obtained from (17) and (18). Let

   $$S = \times_{j \in P} S_j : \mathbb{Z} \times \mathbb{C} \to \times_{j \in P} [0, 1].$$

\(^{20}\)We usually assume that $\mu_j$ depends only on the contributions made to party $j$, but in principle $\mu_j$ could also be lowered by contributions made to other parties.

\(^{21}\)Schotfeld (2007) considers a reduced form version of the organization contribution game, in which $\mu_j$ is assumed a $C^2$, concave function with a maximum at the ideal point of the organizations that support party $j$. For the two candidates case (19) provides microfoundations for $\mu_j$. The key is to assume organizations with partisan preferences.

\(^{22}\)Roemer (2001) argues that "there is not, in general, free entry of representatives of classes into parties."
(b) \( L_k : \mathbb{Z} \times \mathbb{C} \to \mathbb{R} \) is the utility function of leader \( k \in V \) given by (19). Let
\[
L = \times_{k \in V} L_k : \mathbb{Z} \times \mathbb{C} \to \times_{j \in V} \mathbb{R}.
\]

3. **Sequence of play:** First all organization leaders announce their campaign contribution functions. The parties then respond and simultaneously select platforms from the policy space \( \mathbb{Z} \). Then, organization leaders observe the profile of platforms and simultaneously implement their campaign contributions. Voters receive their signals and the election is held.

As Grossman and Helpman (1996) note, there are two equilibrium notions appropriate to this game. The first involves a commitment mechanism on the activists, having the effect that their offers, intended to influence the party leaders, are credible. Reputation, for example in a repeated play game, may suffice. On the other hand, once the party leaders have made their policy pronouncements, then without a commitment device, only the electoral effect will be relevant (because of the preferences of the activists for one party over another). Schofield (2006) avoids some of these difficulties by using a reduced form of the activist functions. The solution to this reduced form game is identical to one where the party leaders themselves have induced policy preferences, but still maximize vote shares. (See the policy preference models by Duggan and Fey [2005] and Peress [2010]). In both cases, the solution concept is local subgame perfect Nash equilibrium.

**Definition 11** A strict (weak) local subgame perfect Nash equilibrium of the stochastic spatial model \( \Gamma_{\text{end}} = (P, V, \mathbb{Z}, \mathbb{C}, S, L) \) is a profile of party positions \( \mathbf{z}^* \in \mathbb{Z} \) and a profile of contribution functions \( \mathbf{c}^* \in \mathbb{C}^* \) such that:

1. For each political party \( j \in P \) there exists an \( \epsilon \)-neighborhood \( B_\epsilon(z_j^*) \subset \mathbb{Z} \) around \( z_j^* \) such that
\[
S_j (\mathbf{z}^*, \mathbf{c}^*(\mathbf{z}^*)) > (\geq) S_j ((\hat{z}_j, z_{-j}^*), \mathbf{c}^*(\hat{z}_j, z_{-j}^*)) \text{ for all } \hat{z}_j \in B_\epsilon(z_j^*) - \{z_j^*\}.
\]

2a. Under commitment. For each leader \( k \in V \) there is no feasible contribution function \( c_k^* \in \mathbb{C}^* \) such that
\[
L_k (\mathbf{z}', c_k^*(\mathbf{z}'), c_{-k}^*(\mathbf{z}')) > L_k (\mathbf{z}^*, \mathbf{c}^*(\mathbf{z}^*))
\]
where \( \mathbf{z}' \) is such that for all \( j \in P \) there exists an \( \epsilon \)-neighborhood \( B_\epsilon(z_j') \subset \mathbb{Z} \) around \( z_j' \) such that
\[
S_j (\mathbf{z}', c_k^*(\mathbf{z}'), c_{-k}^*(\mathbf{z}')) > (\geq) S_j ((\hat{z}_j, z_{-j}'), c_k^*(\hat{z}_j, z_{-j}'), c_{-k}^*(\hat{z}_j, z_{-j}')) \text{ for all } \hat{z}_j \in B_\epsilon(\hat{z}_j) - \{\hat{z}_j\}.
\]

2b. Under no commitment. For each leader \( k \in V \) and each profile of party positions \( \mathbf{z} \) there is no feasible contribution function \( c_k^* \in \mathbb{C} \) such that
\[
L_k (\mathbf{z}, c_k^*(\mathbf{z}), c_{-k}^*(\mathbf{z})) > L_k (\mathbf{z}, \mathbf{c}^*(\mathbf{z})).
\]

**Remark 1.** If \( B_\epsilon(z_j^*) = B_\epsilon(z_j') = \mathbb{Z} \) and we consider only the weak inequality, then the definition above is just the usual one for a subgame perfect Nash equilibrium.

**Remark 2.** A general proof of existence of Nash equilibrium, and hence subgame perfect Nash equilibrium, can be obtained using Brouwer’s fixed point theorem applied to the function space \( \mathbb{C}^* \), if
we assume that the vote share functions are pseudo-concave and \( C^* \) consists of equicontinuous functions (Pugh, 2002).

Let \( \omega^k \) be a measure of the power of organization \( k \). Let \( \left( \tilde{\gamma}^k, \tilde{\phi}^k \right) = (1 + \omega^k) \left( \phi^k, \phi^k \right) \) be a power adjusted measure of the importance that group \( k \) gives to each policy dimension, and \( \left( \bar{\phi}^k, \bar{\phi}^k \right) = \sum_{h \in V} n_k \left( \tilde{\phi}^k, \tilde{\phi}^k \right) \) the corresponding population averages. Define the \textit{adjusted weighted mean of the ideal policies} \( \bar{z}_m = (\bar{\tau}_m, \bar{\gamma}_m) \)

\[
(\bar{\tau}_m, \bar{\gamma}_m) = \sum_{k \in V} n_k \left( \frac{\tilde{\phi}^k}{\phi^k}, \frac{\tilde{\phi}^k}{\phi^k} \right).
\]

(20)

Note that \( \bar{z}_m \) is an adjusted version of the weighted mean \( z_m \) defined in section 3.1 (in fact if \( \omega^k = \omega \) for all \( k \), then \( \bar{z}_m = z_m \)). The difference is that now better organized groups have a larger weight. Denote \( \bar{z}_m = \times_{j \in P} \bar{z}_m \) the \textit{joint adjusted weighted electoral mean} of the stochastic spatial model.

For purposes of exposition, we can develop the model with just two parties, and illustrate the equilibrium responses of organization leaders and parties. Let us suppose that there are only two parties and that the endogenous valence functions are linear in the contributions and the same for both parties, so that \( \mu_j = \mu \sum_{k \in V} c_{k,j} \). Then, the probability that a voter in group \( k \) votes for party \( j \) rather than for party \( l \neq j \), for \( j = 1, 2 \), is:

\[
\rho_j^k (z, c) = \left[ 1 + \exp \left( v^k_{pol}. (z) - v^k_{pol}. (z) + \lambda_l - \lambda_j + \mu \sum_{k \in V} \left( c_{k,j} - c_{k,l} \right) \right) \right]^{-1}.
\]

(21)

As we noted above, there are two motives for organizations to provide contributions: an \textit{influence motive} and an \textit{electoral motive}. Once the parties have made their policy choices, then the electoral motive persists, but not the influence motive. Unless there is a commitment mechanism, activists need only consider the electoral motive in determining the contribution vector. Hence, if the there is no commitment mechanism, in order to determine optimal contributions after the platform profile \( z = (z_1, z_2) \) is announced, each organization leader maximizes (19) taking \( z = (z_1, z_2) \) as given. The first order solution of this problem is\(^{23}\):

\[
c_{k,j} = \bar{\mu} (z, c) \max \left\{ 0, (n_k)^2 \left[ a_{k,j} v^k_{pol}. (z_j) + b_{k,j} - a_{k,l} v^k_{pol}. (z_l) - b_{k,l} \right] \right\}.
\]

(21)

In this case \( \bar{\mu} (z, c) = \mu \sum_{h \in V} n_h \rho^h_1 (z, c) \left( 1 - \rho^h_1 (z, c) \right) \). Thus (21) implies that if \( a_{k,j} v^k_{pol}. (z_j) + b_{k,j} \neq a_{k,l} v^k_{pol}. (z_l) + b_{k,l} \) then each leader contributes at most to one party. If the equality holds then the leader does not contribute to any party. Adding up the first order conditions of all leaders we obtain the following expression:

\[
\frac{\sum_{k \in V} (c_{k,j} - c_{k,l})}{\bar{\mu} (z, c)} = \sum_{k \in V} (n_k)^2 \left[ a_{k,j} v^k_{pol}. (z_j) + b_{k,j} - a_{k,l} v^k_{pol}. (z_l) - b_{k,l} \right].
\]

(22)

Since, given \( z, \bar{\mu} (z, c) \) only depends on \( \sum_{k \in V} (c_{k,j} - c_{k,l}) \), this expression implicitly gives the equilibrium value of \( \sum_{k \in V} (c_{k,j} - c_{k,l}) \) as a function of \( z \) and other parameters. Then, (21) determines the equilibrium

\(^{23}\)The first order condition gives a unique maximum since, given \( z \), we can make \( L_k \) an strictly concave function of \( c_k \). The reason is that we can always find values of \( a_{k,j} \) and \( b_{k,j} \) small enough such that the quadratic cost of collecting the contributions prevails and \( L_k \) becomes an strictly concave function of \( c_k \).
contribution functions. Let \( c^*_k : \mathbb{R}^p \to \mathbb{R}^p \) be the no commitment equilibrium contribution function of organization \( k \), and let \( c^* = \times_{k \in V} c^*_k \). Define
\[
C^*_{j-1}(z) = \sum_{k \in V} \left( c^*_{k,j}(z) - c^*_{k,l}(z) \right).
\]

Parties determine their optimal policy positions with respect to such a profile of no-commitment contribution functions. The problem for party \( j \) is to maximize \( S_j(z) = S_j(z, c^*(z)) \). Since \( S_j(z) \) only involves \( C^*_{j-1}(z) \) and \( C^*_{j-1} \) is a differentiable function of \( z \), \( S_j \) is also a differentiable function of \( z \). Hence we can again use calculus to solve each party problem. The first order necessary condition for party \( j \) is given by:
\[
DS_j(z) = -2 \sum_{k \in V} n_k \rho^k_j(z) \left( 1 - \rho^k_j(z) \right) \begin{pmatrix}
\phi^k_j \left( \tau_j - \tau^k \right) - \frac{\mu}{2} \frac{\partial C^*_{j-1}(z)}{\partial \gamma_j} \\
\phi^k_j \left( \gamma_j - \gamma^k \right) - \frac{\mu}{2} \frac{\partial C^*_{j-1}(z)}{\partial \tau_j}
\end{pmatrix} = 0.
\]

Here \( \rho^k_j(z) = \left[ 1 + \exp \left( v^k_j \left( z \right) - v^k_j \left( z^j \right) - \mu \left( C^*_{j-1}(z) \right) + \lambda_1 - \lambda_2 \right) \right]^{-1} \). Schofield (2006) obtained a similar expression for this “balance equation” in a reduced form of the activist game. Note that the difference between this expression and (15) lies in the gradient term \( \frac{\mu}{2} D C^*_{j-1}(z) \), which Schofield (2006) called the “marginal activist pull.”

The second order sufficient (necessary) condition is that the matrix \( D^2 S_j(z) \) evaluated at a profile that satisfies the first order condition be negative definite (semidefinite).
\[
D^2 S_j(z) = 2 \sum_{k \in V} n_k \rho^k_j(z) \left( 1 - \rho^k_j(z) \right) \left[ 2 \left( 1 - 2 \rho^k_j(z) \right) W^k B^k_{zz} W^k - \tilde{W}^k \right],
\]
\[
\text{where } \tilde{z}_j = z_j - \frac{\mu}{2} \left( W^k \right)^{-1} D C^*_{j-1}(z), \tilde{W}^k = W^k - \frac{\mu}{2} D^2 C^*_{j-1}(z)
\]
\[
\text{and } W^k = \begin{bmatrix}
\phi^k_j & 0 \\
0 & \phi^k_j
\end{bmatrix}; B^k_{zz} = \begin{bmatrix}
(\tilde{\tau}_j - \tau^k)^2 & (\tilde{\gamma}_j - \gamma^k) (\tilde{\tau}_j - \tau^k) \\
(\tilde{\gamma}_j - \gamma^k) (\tilde{\tau}_j - \tau^k) & (\tilde{\gamma}_j - \gamma^k)^2
\end{bmatrix}.
\]

The following proposition characterizes the equilibrium platforms in this no commitment and two parties example.

**Proposition 2** Consider the no commitment stochastic spatial model \( \Gamma_{end} = \{P, V, Z, C, S, L\} \), with exogenous and endogenous valence. Suppose that there are only two parties, \( F^k \) is the extreme value distribution for all \( k \), and the utility functions \( L_k \) are all concave functions of \( c_k \). Suppose further that \( \mu_j = \mu \sum_{k \in V} c_{k,j} \). There are two cases to consider:

1. Suppose that the leaders of the organizations do not have partisan preferences, but they may vary in their influence ability, that is \( a_{k,j} = a_k \) and \( b_{k,j} = b_k \) for all \( j = 1, 2 \). Then \( \omega^k = \bar{\mu}^2 n_k a_k \) for all \( k \), where \( \bar{\mu} = \mu p_1 \left( z_m \right) \left( 1 - p_1 \left( z_m \right) \right) \). For \( \bar{\mu}^2, \mu p_1 \left( z_m \right) \left( 1 - p_1 \left( z_m \right) \right) \) are the unique profile that simultaneously satisfies the first order condition (23) with both parties proposing the same platform. A sufficient (necessary) condition for \( z_m \) to induce a strict (weak) local subgame perfect Nash equilibrium is that the Hessian matrices, \( D^2 S_j(z_m) \), of both parties evaluated at \( z_m \), be negative definite (semidefinite).
2. On the other hand, assume that the leaders of the organizations have strong partisan preferences, in the following sense: There is a partition \( \{ V_1, V_2 \} \) of \( V \) such that for all \( k \in V_1 \) \( a_{k,1} > a_{k,2} \) and \( b_{k,1} > b_{k,2} \), while for all \( k \in V_2 \) \( a_{k,2} > a_{k,1} \) and \( b_{k,2} > b_{k,1} \). Then, a profile \( \mathbf{z}^* \) that satisfies the first order condition requires that each party be located between the electoral joint mean and the ideal policies of the organizations that support the party. A sufficient (necessary) condition for this profile to induce a strict (weak) local subgame perfect Nash equilibrium is that the Hessian matrices, \( \mathbf{D}^2 S_j (\mathbf{z}^*) \), of both parties evaluated at \( \mathbf{z}^* \), be negative definite (semidefinite). \textbf{Proof:} See the appendix. \( \blacksquare \)

Note, from the second part of this proposition, that the “balance equation” implies that the equilibrium position of each party must involve a balance between the centripetal attraction of the electoral center and the centrifugal force of contributions.\(^{24}\)

4.2 Trade policy under convergence

In section 3.2 we studied the determination of trade policy under the assumptions that political competition is purely electoral and parties’ platforms converge. The idea behind this model is a situation in which the electoral franchise is extended to the whole population and groups do not have any extra power to influence policy besides elections. In general, this would not be an accurate representation for at least some countries and some periods of history. Introducing organizations other than political parties allows us to capture an additional source of political power created by how willing each group of voters is to provide contributions in exchange for preferable policies.

Consider a situation with only two parties, in which all activist leaders do not have partisan preferences, and the Hessian matrices of both parties evaluated at \( \bar{z}_m \) are negative definite. Then Proposition 2 (case 1) implies that the political equilibrium outcome is given by the adjusted weighted electoral mean \( \bar{z}_m = (\bar{r}_m, \bar{\gamma}_m) \). This means that the more organized a group is, measured by \( \omega^k \), the more impact the group has in the equilibrium outcome. Therefore, organizations can either moderate or reinforce the conclusions under the assumption of no organizations, depending on which group is able to increase its influence through campaign contributions. For instance, a land rich economy (with structure 1) can be even closer to free trade if the landowner elite has relatively more lobby power than workers, and the nascent industrial capitalists. Alternatively, the landed elite in a moderately land abundant economy, but with a relatively important manufacturing industry (as in an economy with structure 2), can oppose the protectionist propensity of capitalists and workers, using its lobby power. It will be able to do this until such time as capitalists and workers build their own organizations and lobby power.

Thus the model suggests a very rich structure of institutional and economic path dependence. For example, a powerful landowner elite can maintain the economy very close to free trade, discouraging the growth of the secondary sector, and hence avoiding the emergence of a major protectionist force formed by capitalists and workers. It is also possible that an exogenous decrease in the international trade policy would push the economy even further to free trade.\(^{24}\)

\(^{24}\)Schofield (2006) noted this feature of a simple stochastic model involving activists where \( \mathbf{\mu} \) was a concave function of contributions, and suggested that the Hessian term, \( \mathbf{D}^2 \mathbf{\mu} \), could be assumed to have negative eigenvalues of sufficient magnitude so as to induce a pure strategy Nash equilibrium. It would be attractive if this simple model could be extended to a more complex game where organization and party leaders bargain over contributions, as suggested in Schofield and Miller (2007). Such a formal analysis is quite difficult except under simplifying assumptions such as used in this Proposition, but it would also provide a model of activist influence as discussed in Grossman and Helpman (1996).
terms of trade leads to a sufficient growth in the secondary sector, which turns workers in the tradable sector into a protectionist force. The lobby power of landowners and service workers can offset this protectionist impulse for some time. Eventually capitalists and "tradable" workers counterbalance this force by building their own lobby power and creating a more protectionist equilibrium.

Once the economy is in a protectionist equilibrium, landlords and service workers may try to respond by defranchising workers in the tradable sector and suppressing their organizations. Eventually workers in tradable industries will switch to become supporters of free trade. Hence, it is very natural to imagine exogenous and endogenous switches between structures 1 and 2. It is much more complicated to picture this kind of switch in a capital abundant economy, since all groups, except landlords, prefer either free trade or a very moderate protectionism.

In summary, if the introduction of organizations increases the power of the owners of the factor specific to the exporting industry and/or service workers, then the equilibrium trade policy comes closer to free trade. If it increases the power of the owners of the factor specific to the industry that competes with the imports or of workers in the tradable industries, the equilibrium trade policy becomes more protectionist.

4.3 Economic structure, political power and convergence

The way activists influence the convergence coefficients is subtle. Again, assume that there are only two parties and activist leaders do not have partisan preferences (case 1 in proposition 2), then it is possible that convergence is more or less likely with activists than without them. The reason is that the endogenous components of valence have an ambiguous effect on the Hessian matrices of both parties evaluated at $z_m$. On the other hand, if activist leaders have partisan preferences (case 2 in proposition 2), campaign contributions constitute an unambiguous centrifugal force, inducing each party to trade off the electoral mean and the ideal position of the organizations that support the party.

4.4 Trade policy under divergence

Consider a situation with two political parties. Party 1 receives contributions from organizations $k = F_X, F_Y, F_N$ while party 2 receives contributions from organization $L$. Let $z_j^* = (\tau_j^*, \gamma_j^*)$ be the equilibrium platform of party $j = 1, 2$. Regardless the structure of the economy, in equilibrium, party 1 offers a lower fraction of government revenue allocated to $G_F$ than the electoral mean, and party 2 offers a higher fraction of government revenue allocated to $G_L$ than the electoral mean. That is $\gamma_2^* > \gamma_m^* > \gamma_1^*$ (proposition 2, case 2). The reason is fairly intuitive. When party 1 is choosing a platform, then in order to maximize campaign contributions it must balance a centrifugal force that pushes it to the electoral center $\tau_m$, and a centripetal force that pushes it to $\gamma^k = 0$, the ideal policy of the organizations that support the party.

The same logic applies to party 2 with $\gamma^L = 1$. The importance of each of these force varies with political parameters. All else equal, the more effective activists leaders are and the more effective contributions are, the more intense is the centripetal force, and thus the further apart $\gamma_2^*$ and $\gamma_1^*$ will be. Furthermore, ceteris paribus, the higher is the exogenous valence of a party the closer it is to the electoral mean.

The structure of the economy has, however, an important effect on $\tau_j^*$. If the economy has either structure 1 or 3, the ideal export tax rate of workers in the tradable industries $\tau^L$ tends to be very close to the electoral mean $\tau_m$. If the influence ability of the organizations $k = F_X, F_Y, F_N$ does not vary
too much, it is also the case that the weighted ideal export tax rate of these groups is also very close to \( \tau_m \). Therefore, \( \tau_1^* \approx \tau_2^* \approx \tau_m \), and parties’ platform do not have a significant variation in terms of the proposed trade policy. On the other hand, if the economy has structure 2, and the fraction of the owners of factor \( F_Y \) in the population is not very high, so \( \tau^* > \tau_m \), which implies that \( \tau_2^* > \tau_m \). Moreover, if the influence ability of organizations \( k = F_X, F_Y, F_N \) does not vary too much, it is also the case that the weighted ideal export tax rate of these groups must be lower than \( \tau_m \), which implies that \( \tau_1^* < \tau_m \). Therefore, \( \tau_2^* > \tau_m > \tau_1^* \), and parties’ platform differ significantly in terms of the proposed trade policy. Recall also that \( \tau_m \) is higher for an economy with structure 2 than for an economy with structures 1 or 3. Hence, party 2 offers a highly protectionist policy, while party 1 proposes a relatively moderate one.

Summing up, for an economy with structures 1 or 3, both parties tend to propose a very similar and moderate trade policy, while they sharply differ in their budget proposal. Party 1 offers more \( G_F \) and party 2 more \( G_L \). Most of the political conflict is about the budget allocation dimension.

On the other hand, for an economy with structure 2, parties tend to differ in both dimensions. Party 1 offers a moderate trade policy and more \( G_F \), while party 2 offers a highly protectionist trade policy and more \( G_L \). There is political conflict in both dimensions of trade policy and budget allocation.

Finally, note that for an economy with structures 1 and 3, the efficiency of the economy does not significantly vary when there is a change in the party that wins the election, since both parties propose similar trade policies. Distributional conflict mainly occur in the budget allocation, which does not affect the efficiency of the economy. However, for an economy with structure 2, party rotation induces significant changes in the efficiency of the economy since each party implements a very different trade policy.

## 5 Concluding remarks

In this paper we have explored the political and economic consequences of the theoretical model of political economy that was presented in sections 2 and 3 of the paper. We have focused our attention on three main issues. First, we have assumed that the sufficient conditions for policy convergence are satisfied and we have characterized the equilibrium outcome. We have stressed the role of the economic structure in the political equilibrium, and have shown how trade policy is affected by changes in political institutions, such as the lobby power of each group, as well as by economic shocks in the international terms of trade. The model also suggests that path dependent trade policies can emerge as the equilibrium response of the political game to temporary changes in the economic environment and political institutions.

Second, we have discussed the convergence conditions. In particular we have studied how likely it is that an economic structure induces policy convergence. Here the emphasis has been on policy stability, rather than on comparing the equilibrium levels of protection induced by different economic and political structures. This is a point that has not been emphasized in the traditional literature of the political economy of international trade. However, we think it is a relevant one, because high volatility and sudden changes in trade policies have been considered important impediments to growth in many developing countries.

Third, we have studied and interpreted the political equilibrium under divergence. In particular, we have shown that there can exist a political equilibrium in which there is a positive probability of a "populist" outcome with a high level of protection and more public goods for unskilled workers. In addition there can exist a "middle class" outcome with a relatively lower level of protection and more
public goods for specific factors. We interpret this result to mean that, in equilibrium, society can switch from one of these outcomes to the other.

Fourth, globalization has recently been a powerful force in bringing about economic convergence across many countries (see O'Rourke and Williamson [1999]). When there was a backlash against globalization after the 1930 crisis, the result was economic divergence. For many developing countries, this backlash lasted for almost 50 years. Today, there is a persistent fear of a repeat of the past, that the current economic crisis will again induce a backlash against world market integration. Though this is possible, our analysis suggest that the risk of it is less likely than it was eighty years ago. The main reason is the growth of the service economy through world. As we have shown, the development of the non-tradeable sector in the economy plays a key role in political alignments, since service workers push the political equilibrium toward the ideal position of the relative abundant factor in the economy. Therefore, they act as a moderating force against the protectionist tendency.

To conclude, we return to the example of Argentina mentioned in the introduction. It is fair to say that at the beginning of the 20th century Argentina’s factor endowment resembled what we denoted here as a specialized natural resource-rich economy. However, during the interwar period; trade opportunities and the terms of trade worsened and these triggered an industrialization process. This accelerated with the world depression during the 1930s and the Second World War. As a result, Argentina started the second half of the 20th century with a very different economic configuration. Industrialization had come a long way, bringing about what we have called a diversified natural resource-rich economy (see Galiani and Somaini [2009]). These new economic conditions also changed the political equilibrium; urban workers employed in the manufacturing sector and industrialists were the major social actors and they demanded a deepening of the industrialization process. This took the economy close to autarky.

Indeed, the pre 1930 Argentine society remained on the whole flexible, and social mobility was about as high as in other countries of recent settlement. The majority of the elite, although wealthy and powerful, remained attached to a liberal ideology until at least the 1920s, as witnessed by the educational system (see Galiani et al. [2008]). It is not inconceivable that a few more decades of an expanding world economy would have induced acceleration in the growth of urban leadership which could have reconciled the aspirations of urban workers, entrepreneurs, and rural masses with a gradual decline in rural exportable commodities. Yet such a balancing act, even under prosperous conditions, was difficult in Argentina. The main problem that arose was that policies which were best from the viewpoint of economic efficiency (e.g., free, or nearly free, trade) generate an income distribution favorable to the owners of the relatively most abundant factor of production (land). This strengthens the position of the traditional elite. In Argentina, contrary to what occurred in the United States or Britain, by the end of Second World War, what was efficient was not popular (Diaz Alejandro [1970]). Once workers voted on a large scale for the first time in 1946, an urban-rural cleavage developed under the leadership of Peron. This coalition not only shifted trade policy but it also significantly modified the distribution of public expenditures towards the low-income class. In the 21st century, though Argentina still has a diversified natural resource-rich economic structure, the rise of the service economy has debilitated the supremacy of the ‘populist’ coalition and its policy can no longer be viewed in terms of an urban-rural cleavage (see Galiani and Somaini [2009]). At present, either a coalition of the rural sector and the urban middle class nor a coalition of unskilled workers and the industrialist (that compete with imports) can win a democratic election in Argentina.

This brief sketch indicates how the electoral preferred degree of protection can be transformed as
a result of essentially political changes in the balance of power between landed and capital elites, in
coalition with different elements of enfranchised labor.

References

[1] Albornoz, Facundo, Sebastian Galiani, and Daniel Heymann, "Investment and expropriations under

[2] Acemoglu, Daron, and James A. Robinson, Economic Origins of Dictatorship and Democracy (Cam-
bridge: Cambridge University Press, 2005).

American Economic Review, 98 (2008), 808-842.


(Heidelberg: Springer, 2005).


[8] Cantón, Dario, Materiales para el estudio de la sociología política en la Argentina, Mimeo, Buenos
Aires, 1968.


[12] Duggan John and Fey Mark Fey, "Electoral Competition with Policy-Motivated Candidates" Games
and Economic Behavior 51 (2005), 490-522.


6 Appendix

In this appendix we prove some results about the economic model, and convergence conditions for the convenience of the reader.

6.1 Proof of Lemma 1: conditions for comparative advantage in industry X

In this subsection we solve the model under autarky and we deduce the tax rate that induces autarky. Under autarky the market clearing conditions in goods X and N are \( Q_X = C_X \) and \( Q_Y = C_Y \). Merging these two expression with households consumption decisions we have \( \frac{p_X Q_X}{\beta_X} = \frac{p_Y Q_Y}{\beta_Y} \). From equation (1)

\[
\frac{p_X (1 - \alpha_X) \hat{Q}_X}{(1 - l_Y)\alpha X} = \frac{p_Y (1 - \alpha_Y) \hat{Q}_Y}{(l_Y)\alpha Y}
\]

Let’s call by \((l_Y)_{aut} \) and \((\frac{p_X}{p_Y})_{aut} \) the equilibrium values of \( l_Y \) and \( \frac{p_X}{p_Y} \) under autarky. From the previous expressions we obtain

\[
(l_Y)_{aut} = \frac{\beta_Y (1 - \alpha_Y)}{\beta_Y (1 - \alpha_Y) + \beta_X (1 - \alpha_X)}, \quad \left(\frac{p_X}{p_Y}\right)_{aut} = \frac{\Psi}{\Omega}
\]

where \( \Psi = \frac{\Delta Y (\hat{Y}_X)\alpha Y (\hat{L}_X)\alpha X}{\hat{X}_X (\hat{F}_X)\alpha X} \), and \( \Omega = \frac{(\beta_Y)\alpha Y (1-\alpha_X)\alpha_X (1-\alpha_M)\alpha_M \beta_Y (1 - \alpha_Y) + \beta_X (1 - \alpha_X)\alpha X - \alpha Y}{(\beta_X)\alpha X (1-\alpha_M)\alpha_M \beta_Y (1 - \alpha_Y) + \beta_X (1 - \alpha_X)\alpha X - \alpha Y} \). Therefore the economy has a comparative advantage in industry X if and only if \( \frac{p_X}{p_Y} > \left(\frac{p_X}{p_Y}\right)_{aut} \), or in terms of parameters \( \Psi < \Gamma \frac{p_X}{p_Y} \), and a comparative advantage in industry Y if and only if the reverse inequality holds, which completes the proof of lemma 1.

6.2 Proof of Lemma 2: specific factors

Note that \( \eta_{w_k/CPI, \tau} = \eta_{w_k/CPI, \tau} \eta_{p_X, \tau} \). Since \( \eta_{p_X, \tau} = -\frac{\tau}{1-\tau} < 0 \) for all \( \tau \in (0,1) \) we focus on the sign of \( \eta_{w_k/CPI, p_X} \).

Suppose that the economy has a comparative advantage in industry X, that is \( \Psi < \Omega \frac{p_X}{p_Y} \). The elasticity of \( w_{FX}/CPI \) w.r.t. \( p_X \) is given by

\[
\eta_{w_{FX}/CPI, p_X} = \eta_{w_{FX}, p_X} - \beta_X - \beta_N \eta_{p_N, p_X} \\
= 1 + (1 - \alpha_X) \eta_{X, p_X} - \beta_X - \beta_N \eta_{p_N, p_X} \\
= 1 + (1 - \alpha_X) \eta_{X, p_X} - \beta_X - \beta_N \left( \frac{\beta_X}{\beta_X + (1-\tau)\beta_Y} + (1 - \alpha_X) \eta_{X, p_X} \theta_{X, \tau} \right) \\
= \left( 1 - \beta_X - \frac{\beta_N \beta_X}{(1-\tau)\beta_Y + \beta_X} \right) + (1 - \alpha_X) \eta_{X, p_X} (1 - \beta_X \theta_{X, \tau}),
\]

The first line uses the definition of \( \eta_{w_{FX}/CPI, p_X} \); the second line computes \( \eta_{w_{FX}, p_X} \) from the profit maximization of firms in industry X; the third line uses (5); and the fourth line is just a rearrangement.
of terms. The final expression is positive for all \( \tau \in [0, \tau_{aut.}] \); which implies that \( (w_{F_X}/\text{CPI}) \) is strictly decreasing in \( \tau \) for all \( \tau \in [0, \tau_{aut.}] \).

The elasticity of \( w_{F_X}/\text{CPI} \) w.r.t. \( p_X \) is given by

\[
\eta_{w_{F_X}/\text{CPI},p_X} = \eta_{w_{F_X},p_X} - \beta_X - \beta_N \eta_{p_N,p_X} \\
= (1 - \alpha_Y) \eta_{l_Y,p_X} - \beta_X - \beta_N \eta_{p_N,p_X}.
\]

The second line computes \( \eta_{w_{F_X},p_X} \) from the profit maximization of firms in industry \( Y \). The expression is negative for all \( \tau \in [0, \tau_{aut.}] \) because all the terms are negative; which implies that \( (w_{F_X}/\text{CPI}) \) is increasing in \( \tau \) for all \( \tau \in [0, \tau_{aut.}] \).

The elasticity of \( w_{F_X}/\text{CPI} \) w.r.t. \( p_Y \) is given by

\[
\eta_{w_{F_X}/\text{CPI},p_Y} = \eta_{w_{F_X},p_Y} - \beta_Y - \beta_N \eta_{p_N,p_Y} \\
= (1 - \alpha_X) \eta_{l_Y,p_Y} - \beta_Y - \beta_N \eta_{p_N,p_Y}.
\]

The first line uses the definition of \( \eta_{w_{F_X},p_Y} \); the second line computes \( \eta_{w_{F_X},p_Y} \) from the profit maximization of firms in industry \( X \). The final expression is negative for all \( \tau \in [0, \tau_{aut.}] \); which implies that \( (w_{F_X}/\text{CPI}) \) is strictly increasing in \( \tau \) for all \( \tau \in [0, \tau_{aut.}] \).

The elasticity of \( w_{F_Y}/\text{CPI} \) w.r.t. \( p_Y \) is given by

\[
\eta_{w_{F_Y}/\text{CPI},p_Y} = \eta_{w_{F_Y},p_Y} - \beta_Y - \beta_N \eta_{p_N,p_Y} \\
= 1 + (1 - \alpha_Y) \eta_{l_Y,p_Y} - \beta_Y - \beta_N \eta_{p_N,p_Y} \\
= 1 + (1 - \alpha_Y) \eta_{l_Y,p_Y} - \beta_Y - \beta_N \left( \frac{\beta_Y}{\beta_Y + (1 - \tau) \beta_X} + (1 - \alpha_Y) \eta_{l_Y,p_Y} \theta_Y \tau \right) \\
= \left( 1 - \beta_X - \frac{\beta_N \beta_Y}{(1 - \tau) \beta_X + \beta_Y} \right) + (1 - \alpha_Y) \eta_{l_Y,p_Y} \left( 1 - \beta_N \theta_Y \tau \right).
\]

\(^{25}\)This only applies for an economy not specialized in \( X \), that is \( F_Y > 0 \).
The first line uses the definition of $\eta_{w_Y/CPI_Y}$; the second line computes $\eta_{w_Y,p_Y}$ from the profit maximization of firms in industry $Y$; the third line uses (5); and the fourth line is just a rearrangement of terms. The final expression is positive for all $\tau \in [0, \tau_{aut.}]$, which implies that $(w_{FY}/CPI)$ is strictly decreasing in $\tau$ for all $\tau \in [0, \tau_{aut.}]$.

The elasticity of $w_{FN}/CPI$ w.r.t. $p_Y$ is given by

$$
\eta_{w_{FN}/CPI,Y} = \eta_{w_Y,p_Y} - \beta_Y - \beta_N \eta_{p_N,p_Y} = (1 - \beta_N) \eta_{p_N,p_Y} - \beta_Y = (1 - \beta_N) \left( \frac{\beta_Y}{(1 - \tau) \beta_X + \beta_Y} + (1 - \alpha_Y) \eta_{l_Y,p_Y} \theta_Y \tau \right) - \beta_Y = \left( \frac{(1 - \beta_N) \beta_Y}{(1 - \tau) \beta_X + \beta_Y} - \beta_Y \right) + (1 - \beta_N) (1 - \alpha_Y) \eta_{l_Y,p_Y} \theta_Y \tau.
$$

The first and second line are evident; the third line uses (5). The final expression is positive for all $\tau \in (0, \tau_{aut.}]$ and zero for $\tau = 0$, which implies that $(w_{FN}/CPI)$ is decreasing in $\tau$ for all $\tau \in [0, \tau_{aut.}]$.

**6.3 Proof of Lemma 3: mobile factor**

Suppose that the economy is specialized in $X$, that is $\Psi = 0$. Then

$$
\eta_{w_L/CPI,X} = \eta_{w_X,p_X} - \beta_X - \beta_N \eta_{p_N,p_X} = 1 - \beta_X - \beta_N \eta_{p_N,p_X} = 1 - \beta_X - \beta_N \left( \frac{\beta_X}{\beta_X + (1 - \tau) \beta_Y} \right).
$$

The second line computes $\eta_{w_L,p_X}$ from profit maximization in industry $X$, and the third line uses (5). The final expression is clearly positive, which implies that $(w_L/CPI)$ is decreasing in $\tau$ for all $\tau \in [0, \tau_{aut.}]$.

Suppose that the economy has a comparative advantage in $X$, but it is not specialized, that is $0 < \Psi < \frac{\Gamma_{p_X}^X}{p_Y}$. Then

$$
\eta_{w_L/CPI,X} = \eta_{w_L,p_X} - \beta_X - \beta_N \eta_{p_N,p_X} = -\alpha_Y \eta_{l_Y,p_X} - \beta_X - \beta_N \eta_{p_N,p_X} = \frac{\alpha_X}{\alpha_Y (1 - l_Y)} - \beta_X - \beta_N \left( \frac{\beta_X}{\beta_X + (1 - \tau) \beta_Y} \right) + (1 - \alpha_X) \eta_{l_X,p_X} \theta_X \tau \\
\leq \frac{\alpha_Y (1 - l_Y) + \alpha_X l_Y}{\alpha_Y (1 - l_Y) + \alpha_X l_Y} - \beta_X - \beta_N \left( \frac{\beta_X}{\beta_X + \beta_Y} \right) \\
\leq \frac{\alpha_Y (1 - l_Y (0)) + \alpha_X l_Y (0)}{\alpha_Y (1 - l_Y (0)) + \alpha_X l_Y (0)} - \left( \frac{\beta_X}{\beta_X + \beta_Y} \right) \\
= \frac{\alpha_Y (1 - l_Y (0)) + \alpha_X l_Y (0)}{\alpha_Y (1 - l_Y (0)) + \alpha_X l_Y (0)} - \left( \frac{\beta_X}{\beta_X + \beta_Y} \right).
$$

The second line computes $\eta_{w_L,p_X}$ from the profit maximization of firms in industry $Y$; the third line uses (4) and (5); the fourth line employs two facts: $\frac{\beta_X}{\beta_X + (1 - \tau) \beta_Y}$ is increasing in $\tau$ and $(1 - \alpha_X) \eta_{l_X,p_X} \theta_X \tau \geq 0$.
and the fifth line uses the fact that \( \frac{\alpha_Y(1-L_y)}{\alpha_Y(1-L_y)+\alpha_XL_X} \) is decreasing in \( L_y \). Inequalities in lines four and five are strict if \( \tau > 0 \). The final expression is nonpositive if and only if \( L_y(0) \geq \frac{\beta_Y\alpha_Y}{\beta_X\alpha_Y+\beta_X\alpha_X} \), which is true if and only if \((1-L_y(0))^{\alpha_X} \leq \frac{\beta_X^{\alpha_X}}{\beta_X^{\alpha_X}+\beta_X^{\alpha_X}}\left(1-L_y(0)\right)^{\alpha_X}\), or which is equivalent due to equation (1), if and only if \((\frac{\alpha_X}{1-\alpha_Y})^{\alpha_Y}\left(1-\alpha_X\right)^{\alpha_X} \leq \Psi < \Omega_{\frac{p_X}{p_Y}}^{\alpha_Y}\). Therefore, if \((\frac{\alpha_X}{1-\alpha_Y})^{\alpha_Y}\left(1-\alpha_X\right)^{\alpha_X} \leq \Psi < \Omega_{\frac{p_X}{p_Y}}^{\alpha_Y}\) the sign of \( \eta_{w_L/CPI}p_X \) always depends on \( \tau \). For example, for \( \tau = 0 \) we have

\[
\eta_{w_L/CPI}p_X(\tau_{aut}) = \frac{\alpha_Y(1-L_y(0))}{\alpha_Y(1-L_y(0))+\alpha_XL_X(0)} - \beta_X + \frac{\beta_X}{\beta_X+(1-\alpha_X)\eta_{l_X}p_X}\theta_X\tau_{aut}
\]

which is positive. However, for \( \tau = \tau_{aut} \) we have

\[
\eta_{w_L/CPI}p_X(\tau_{aut}) = \frac{\alpha_Y(1-L_y(\tau_{aut}))}{\alpha_Y(1-L_y(\tau_{aut}))+\alpha_XL_X(\tau_{aut})} - \beta_X + \frac{\beta_X}{\beta_X+(1-\alpha_X)\eta_{l_X}p_X}\theta_X\tau_{aut}
\]

The second line uses two facts: \( \frac{\beta_X}{\beta_X+(1-\alpha_X)\eta_{l_X}p_X} \) is increasing in \( \tau \) and \( (1-\alpha_X)\eta_{l_X}p_X\theta_X\tau \geq 0 \); and the third line uses the expression of \( l_Y(\tau_{aut}) \). The final expression is negative.

Finally, suppose that the economy has a comparative advantage in industry \( Y \), that is \( \Psi > \Omega_{\frac{p_X}{p_Y}}^{\alpha_Y}\). We prove that \((w_L/CPI)\) is decreasing in \( \tau \) for all \( \tau \in [0, \tau_{aut}] \) if the following two conditions hold

C1: \( \alpha_X \geq \max\left\{ \frac{\beta_X}{\beta_X(1-\alpha_Y)}, \frac{\beta_X+\alpha_X\beta_Y}{\alpha_X\beta_X+\alpha_X\beta_Y} \right\} \)

C2: \( \Omega_{\frac{p_X}{p_Y}}^{\alpha_Y} < \Psi < \frac{1}{1-\tau_{aut}}\Omega_{\frac{p_X}{p_Y}}^{\alpha_Y} \), where \( \tau_{aut} = \left(1+\frac{1}{\frac{\beta_Y}{\beta_X}}\right) - \sqrt{\left(1+\frac{1}{\frac{\beta_Y}{\beta_X}}\right)^2 - 1} \)

Step 1: C1 \( \Rightarrow (1-\alpha_Y)\eta_{l_Y}p_Y \leq 1 \)

Since \( \eta_{l_Y}p_Y \) is an increasing function of \( \tau \), \( (1-\alpha_Y)\eta_{l_Y}p_Y < (1-\alpha_Y)\eta_{l_Y}p_Y(\tau_{aut}) \). Due to C1

\( \alpha_X \geq \frac{\beta_X}{\beta_X(1-\alpha_Y)+\beta_Y(1-\alpha_Y)} \), and hence \( (1-\alpha_Y)\eta_{l_Y}p_Y(\tau_{aut}) = \frac{\alpha_Y\beta_X(1-\alpha_X)+\alpha_X\beta_Y(1-\alpha_Y)}{\alpha_Y\beta_X(1-\alpha_X)+\alpha_X\beta_Y(1-\alpha_Y)} \leq 1 \).

Step 2: \( (1-\alpha_Y)\eta_{l_Y}p_Y \leq 1 \) and C2 \( \Rightarrow \eta_{p_X,p_Y} \leq 1 \)

\[
\eta_{p_X,p_Y} = \frac{\beta_Y}{(1-\tau)\beta_X+\beta_Y} + (1-\alpha_Y)\eta_{l_Y}p_Y\theta_Y\tau
\]

\[
\leq \frac{\beta_Y}{(1-\tau_{aut})\beta_X+\beta_Y} + \tau_{aut}.
\]
The first line is just the expression of \( \eta_{w_L/CPI,\tau} \); and the second line uses the following facts: \( \frac{\beta_Y}{(1-\tau)\beta_X+\beta_Y} \) is increasing in \( \tau \), \( (1 - \alpha_Y) \eta_{IY,\tau} \leq 1 \) by assumption and \( \theta_Y \leq 1 \). Furthermore, \( \frac{\beta_Y}{(1-\tau_{aut})\beta_X+\beta_Y+\tau_{aut}} \leq 1 \) if and only if \( \tau_{aut} \leq \tau_{aut} = (1 + \frac{1}{2} \beta_Y/N) - \sqrt{(1 + \frac{1}{2} \beta_Y/N)^2 - 1} \).

Step 3: \( \eta_{p_N,\tau} \leq 1 \) and C1 \( \Rightarrow \eta_{w_L/CPI,\tau} > 0 \)

\[
\eta_{w_L/CPI,\tau} = \eta_{w_L,\tau} - \beta_Y - \beta_N \eta_{p_N,\tau}
= \alpha_X \eta_{I_X,\tau} - \beta_Y - \beta_N \eta_{p_N,\tau}
= \frac{\alpha_X l_Y}{\alpha_Y (1 - l_Y) + \alpha_X l_Y} - \beta_Y - \beta_N \eta_{p_N,\tau}
\geq \frac{\alpha_X l_Y (\tau_{aut})}{\alpha_Y (1 - l_Y (\tau_{aut})) + \alpha_X l_Y (\tau_{aut})} - \beta_Y - \beta_N
\geq \frac{\alpha_X \beta_Y (1 - \alpha_Y)}{\alpha_Y \beta_X (1 - \alpha_X) + \alpha_X \beta_Y (1 - \alpha_Y)} - \beta_Y - \beta_N
\]

The second line computes from the profit maximization of firms in industry \( X \); the third line uses (4); the fourth line employs the assumption \( \eta_{p_N,\tau} \leq 1 \) and the fact that \( \frac{\alpha_X l_Y}{\alpha_Y (1 - l_Y) + \alpha_X l_Y} \) is decreasing in \( \tau \). The final expression is positive since C1 implies \( \alpha_X \geq (\beta_Y + \beta_N)\alpha_Y \). Therefore, if C1 and C2 hold \( (w_L/CPI) \) is decreasing in \( \tau \) for all \( \tau \in [0, \tau_{aut}] \), which completes the proof of the lemma.

### 6.4 Proof of lemma 4: Ideal policies

Since \( v^k (\tau, \gamma) \) is a continuous function and the policy space \( Z \) is a compact set, a global maximum \( (\tau^k, \gamma^k) \) exists. Since \( v^k (\tau, \gamma) \) is strictly increasing in \( \gamma \) for \( k = L \) and strictly decreasing in \( \gamma \) for \( k = F_X, F_Y, F_N \) we have \( \gamma^k = 0 \) for \( k = F_X, F_Y, F_N \) and \( \gamma^L = 1 \). The ideal tax rate \( \tau^k \) must be interior because for \( \tau = 0 \) and \( \tau = \tau_{aut} \) government revenue is zero and \( H'(0) \to -\infty \). Therefore, the derivative of \( v^k (\tau, \gamma) \) with respect to \( \tau \) evaluated at \( (\tau^k, \gamma^k) \) must be equal to zero, or which is equivalent \( \tau^k \) must satisfies:

\[
\eta_{w_k/CPI,\tau} \frac{w_k^*}{\eta_k} + RH' \left( \frac{R}{CPI} \right) \eta_{R/CPI,\tau} = 0
\]

Suppose an economy with structure 1. It is not difficult to verify from the proof of lemmas 2 and 3 that \( \eta_{w_F,\tau} > 0 \), and \( \frac{w_{FX}}{p_{FX}} > \frac{w_{FN}}{p_{FN}} > 0 \), \( \eta_{w_F,\tau} > 0 \), and \( \eta_{w_F,\tau} > 0 \). Since \( H'(0) \to -\infty \) and \( \max \left\{ \frac{w_{FX}}{p_{FX}}, \frac{w_{FY}}{p_{FY}} \right\} \geq \frac{w_{FX}}{p_{FX}} \geq \frac{w_{FL}}{p_{FL}} \), the previous expression implies that \( \tau^F_X < \tau^F_N < \tau^L < \tau_{max} < \tau^F_Y \). An analogous argument applies for an economy with structure 3, just reversing the roles of \( F_X \) and \( F_Y \). For an economy with structure 2, again it is not difficult to verify that \( \eta_{w_F,\tau} > 0 \), and \( \eta_{w_F,\tau} < 0 \), which implies that \( \tau^F_X < \tau^F_N < \tau_{max} < \tau^L < \tau^F_Y \).

### 6.5 Proof of propositions 1

As we have already shown, the joint weighted electoral mean, \( z_m \), satisfies the first order condition for local equilibrium for all parties (15). Hence, in order to verify that \( z_m \) is a strict local Nash equilibrium,
we only need to check whether the Hessian matrix of each party evaluated at \( z_m \) is negative definite. To prove that \( c(\Gamma_{exo}) < 1 \) is sufficient for \( D^2 S_j(z_m) \) to be negative definite for all \( j \in P \), we proceed as follows. We have defined the characteristic matrix as \( H_j(z_m) = \sum_{k \in V} n_k (A_j W^k B^{k}_{z_m} W^k - W^k) \). Then, the Hessian matrix of party \( j \) evaluated at \( z_m \) is given by:

\[
D^2 S_j(z_m) = 2\rho_j(z_m) \left( 1 - \rho_j(z_m) \right) H_j(z_m).
\]

Since \( 2\rho_j(z_m) \left( 1 - \rho_j(z_m) \right) \) is a positive constant, \( D^2 S_j(z_m) \) is negative definite (semidefinite) if and only if \( H_j(z_m) \) is negative definite (semidefinite). The trace of \( H_j(z_m) \) is given by

\[
\text{Tr} (H_j(z_m)) = \sum_{k \in V} n_k \text{Tr} \left( A_j W^k B^{k}_{z_m} W^k - W^k \right)
= A_j \sum_{k \in V} n_k \text{Tr} \left( W^k B^{k}_{z_m} W^k - \sum_{k \in V} n_k \text{Tr} \left( W^k \right) \right)
= \left[ \frac{A_j}{A_1} \text{det}(\Gamma_{exo}) - 1 \right] \sum_{k \in V} n_k (\phi^k - \phi^k_{\gamma})
\]

Since parties are ordered according to their valences \( A_1 \geq \ldots \geq A_j \geq \ldots \geq A_p \), this implies

\[
\text{Tr} (H_j(z_m)) \geq \ldots \geq \text{Tr} (H_j(z_m)) \geq \ldots \geq \text{Tr} (H_p(z_m)).
\]

Therefore, if \( d(\Gamma_{exo}) < 1 \), then \( \text{Tr}(H_j(z_m)) < 0 \) for all \( j \in P \).

The determinant of \( H_j(z_m) \) is given by

\[
\text{det} (H_j(z_m)) = (A_j)^2 \sum_{k \in V} n_k \left( W^k B^{k}_{z_m} W^k \right)_{11} \sum_{k \in V} n_k \left( W^k B^{k}_{z_m} W^k \right)_{22}
- (A_j)^2 \left[ \sum_{k \in V} \left( W^k B^{k}_{z_m} W^k \right)_{21} \right]^2
+ \left( \sum_{k \in V} n_k \phi^k \right) \left( \sum_{k \in V} n_k \phi^k_{\gamma} \right) \left[ 1 - \frac{A_j}{A_1} \text{det}(\Gamma_{exo}) \right]
\]

By the triangle inequality, the sum of the first two terms in this expression for \( \text{det} (H_j(z_m)) \) must be non-negative. Moreover, \( A_1 \geq \ldots \geq A_j \geq \ldots \geq A_p \) implies

\[
\left[ 1 - \frac{A_j}{A_1} \text{det}(\Gamma_{exo}) \right] \geq \ldots \geq \left[ 1 - \frac{A_j}{A_1} \text{det}(\Gamma_{exo}) \right] \geq \ldots \geq [1 - c(\Gamma_{exo})].
\]

Therefore, if \( c(\Gamma_{exo}) < 1 \), then \( \text{det}(H_j(z_m)) > 0 \) for all \( j \in P \).

Since \( d(\Gamma_{exo}) < c(\Gamma_{exo}), \) then \( c(\Gamma_{exo}) < 1 \) implies that \( \text{Tr}(H_j(z_m)) < 0 \), and \( \text{det}(H_j(z_m)) > 0 \) for all \( j \in P \). Thus \( c(\Gamma_{exo}) < 1 \) is a sufficient condition for \( D^2 S_j(z_m) \) to be negative definite for all \( j \in P \). This completes the proof of sufficiency.

For the necessary part, assume that \( z_m \) is a weak local Nash equilibrium. Then the Hessian matrix of each party evaluated at \( z_m \) must be negative semidefinite. This implies \( \text{det}(D^2 S_j(z_m)) \geq 0 \) and \( \text{Tr}(D^2 S_j(z_m)) \leq 0 \) for all \( j \in P \). This is true if and only if \( \text{det}(H_j(z_m)) \geq 0 \) and \( \text{Tr}(H_j(z_m)) \leq 0 \) for all \( j \in P \). \( \text{Tr}(H_1(z_m)) \leq 0 \) if and only if \( d(\Gamma_{exo}) \leq 1 \). If \( d(\Gamma_{exo}) > 1 \), then \( \text{Tr}(H_1(z_m)) \) must be strictly positive, and so one of the eigenvalues of \( H_1(z_m) \) must be strictly positive, violating the weak Nash equilibrium condition. This completes the proof of necessity. 

38
6.6 Proof of proposition 2

Suppose non partisan organizations and that the influence ability of each organization is the same for both parties. Then, from (22) is not difficult to check that if we consider a profile \( z \) such that \( z_1 = z_2 = (\tau, \gamma) \) then: (i) \( C^*_1(z) = C^*_2(z) = 0 \), (ii) \( \rho_1(z) = [1 + \exp(\lambda_2 - \lambda_1)]^{-1} \), and (iii) 
\[
\frac{\partial}{\partial z} D C^*_j(z) = \rho_1(z) (1 - \rho_1(z)) \sum_{k \in V} (n_k)^2 a_{k,j} \begin{pmatrix} \phi^k_1 (\tau - \tau^k) \\ \phi^k_2 (\gamma - \gamma^k) \end{pmatrix}.
\]
Introducing (i)-(iii) into the first order condition (23), and rearranging terms we obtain a system of equations, whose unique solution is the profile \( \bar{z}_m \). Therefore, \( \bar{z}_m \) is the unique profile that simultaneously satisfies the first order condition and predicts parties convergence. A sufficient (necessary) condition for \( \bar{z}_m \) to induce a strict (weak) local maximum for each party is that the Hessian matrices of both parties evaluated at \( \bar{z}_m \), denoted \( D^2 S_j(\bar{z}_m) \), be negative definite (semidefinite). Finally, if \( \bar{z}_m \) induces a strict (weak) local maximum for both parties, then \( \bar{z}_m \) is a strict (weak) local Nash equilibrium of the game \( \Gamma_{end} \). Hence, the parties platforms \( \bar{z}_m \) and the contribution functions \( c^*_k(j) \) form a strict (weak) local subgame perfect Nash Equilibrium, which completes the proof of the first part of the proposition. Now, suppose that each organization is attached to only one specific party. Rearranging terms in the first order condition (23) we obtain a system of equations:

\[
\tau_j = \sum_{k \in V} \frac{\rho^k_j(z) \left( 1 - \rho^k_j(z) \right) n_k \phi^k_\tau}{\sum_{h \in V} \rho^h_j(z) \left( 1 - \rho^h_j(z) \right) n_h \phi^h_\tau} \tau^k + \frac{\mu}{2} \frac{\partial C^*_j(z)}{\partial \tau_j},
\]

(25)

\[
\gamma_j = \sum_{k \in V} \frac{\rho^k_j(z) \left( 1 - \rho^k_j(z) \right) n_k \phi^k_\gamma}{\sum_{h \in V} \rho^h_j(z) \left( 1 - \rho^h_j(z) \right) n_h \phi^h_\gamma} \gamma^k + \frac{\mu}{2} \frac{\partial C^*_j(z)}{\partial \gamma_j},
\]

(26)

We now show that \( z_1 = z_2 \) cannot be a solution of this system. Assume for a moment that \( z_1 = z_2 = (\tau, \gamma) \) is a solution of the system of balance equations, then from (22) 
\[
\frac{\partial}{\partial z} D C^*_j(z) = \rho_1(z) (1 - \rho_1(z)) \sum_{k \in V} (n_k)^2 a_{k,j} \begin{pmatrix} \phi^k_1 (\tau - \tau^k) \\ \phi^k_2 (\gamma - \gamma^k) \end{pmatrix}.
\]
Hence \( D C^*_j(z) \neq D C^*_j(z) \), which due to (25) and (26) implies that \( \tau_1 \neq \tau_2 \) and \( \gamma_1 \neq \gamma_2 \), a contradiction. Therefore there is no profile that at the same time satisfies \( z_1 = z_2 \) and the first order condition (23). From the balance conditions (25)-(26) we observe that the equilibrium position of each party, denoted denoted \( z^*_j \), must be a trade off between the centrifugal force of electoral center, captured by the first terms of the right hand side of (25) and (26), and the centripetal force of contributions, captured by the second terms of right hand side of (25) and (26). Following the same arguments of the first part of the proof a sufficient (necessary) condition for this profile to induce a strict (weak) local Subgame Perfect Nash equilibrium is that the Hessian matrices of both parties evaluated at this profile \( D^2 S_j(z^*) \) be negative definite (semidefinite).