International Cross-Ownership and Strategic Trade Policies

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We analyze a model of strategic trade policies in the presence of international cross-ownership of …rms that are heterogenous both in terms of costs and in terms of extent of foreign ownership. The equilibrium pattern of taxes and subsidies is characterized for any arbitrary cross ownership pro…le, and any number of heterogenous …rms. The equilibrium subsidy (or tax) given to any …rm is shown to depend, in a separable manner, on the …rm's characteristics and on the co-variance of the distribution of cost and foreign ownership accross …rms. An independence theorem is proved concerning the Nash equilibrium of the game between governments: in equilibrium, the pattern of trade, the value of each …rm, and world welfare are independent of the ownership pattern, provided that each …rm is taxed or subsidized by both governments.

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1 Introduction

In the literature on strategic trade policies, it is typically assumed that …rms located in any country are owned by residents of that country. There exists, of course, a literature on multinational. As equity markets are becoming more and more integrated, it seems important to determine the extent to which the pattern of cross-ownership in‡uences the structure and levels of protection that a national government gives to …rms that are domestically based but not wholly owned by domestic residents, as well as the levels of subsidies that it might give to …rms that are based abroad but are partially owned by domestic residents[1]. Naturally, in formulating strategic trade policies, national governments should take into account the international ‡ows of dividends[2].

Bhagwati and Brecher (1980), Brecher and Bhagwati (1981), and Brecher and Findlay (1983) have studied the implications of foreign ownership on trade policy and welfare when …rms are price-takers. Under the assumption of Cournot rivalry, Lee (1990), using a two-country, two-…rm model, showed that if both governments are engaged in a non-cooperative game of import protection and export promotion, then the resulting Nash equilibrium of this policy game displays an interesting property: the pattern of ownership has no e¤ect the equilibrium trade patterns and pro…ts of …rms, and world welfare is a constant. We will refer to this as Lee's "neutrality result". He pointed out (in his footnote 4) that this result can also be obtained if there are a …xed number of identical …rms in each country. Lee's analysis is restricted to the case where only one …rm (the exporting …rm) is subject to tax or subsidy by both governments while the other …rm is neither directly helped nor directly hindered by any government. Furthermore, it is not clear if the neutrality result still holds if there are several heterogenous …rms (in terms of costs, or in terms of extent of foreign ownership) in each country.

In this paper, we take up these issues, in the context of the "third market"

[1]According to Dick (1993, p.228), "approximately one third of total U.S. merchandise imports are manufactured by foreign-based a¢liates of U.S. corporations, while approximately one-…fth of total U.S. merchandise exports are produced by foreign-owned a¢liates operating in the United States."

[2]Dick (1993) estimates that existing levels of US cross ownership reduce the average optimal export subsidy by 47 per cent relative to the Brander and Spencer (1985) value. Levinsohn (1989) has shown that tari¤s and quotas have di¤erent welfare implications in the presence of foreign ownership.

model[3], as in Brander and Spencer (1985), under more general asumptions. Firstly, we introduce a factual asymmetry: in each country we allow an arbitrary number of ex ante non-identical …rms, both in terms of costs and in terms of the extent of foreign ownwership. Secondly, our model takes into account an important logical asymmetry: both governments can directly interfere with any …rm's cost by subsidizing it, even if the …rm is located in a di¤erent country, but neither government can tax …rms that are located in a foreign country. (Recall that domestic and foreign …rms compete in a third market.) Thirdly, since within each country …rms are ex ante asymmetric, governments would want to set …rm-speci…c taxes or subsidies. This is permitted in our model. Fourthly, to take into account the fact that government cannot impose lump-sum tax on consumers, we introduce a parameter in the social welfare function to represent the cost of public funds. In other words, a dollar of tax collected from consumers not only reduces their income by a dollar, but also creates a deadweight loss by distorting consumers' choice. (In this, we are following Neary (1994), who uses this important concept in public …nance in his analysis of strategic trade policies.)

We are therefore dealing with a much more complex model than Lee's. This general formulation enables us to obtain a richer set of results. We show how the structure of equilibrium taxes and subsidies re‡ects the distribution of marginal costs, the pattern of ownership, and the marginal cost of public …nance. Certain separability properties are shown to hold in equilibrium. Tax and subsidy formulas can be neatly described despite the complexity of the model. We are able to derive tax-subsidy fomulas that contain two separate components, one pertaining to a …rm's idiosyncratic properties, and the other relating to the average characteristics shared by all …rms.Our formulas for equilibrium taxes and subsidies contain terms that are easily interpreted, such as the variance of the distribution of marginal costs, and the co-variance between costs and ownership parameters. A generalization of Lee's 'neutrality result' is obtained provided the equilibrium does not involve corner solutions. This neutrality result is robust with respect to asymmetry of costs and of ownership pro…les.

[3] The term 'third market', used by Brander (1995, p.1405) has become a standard description of the model studied by Brander and Spencer (1985). Interference by two governments in an independent third market is not a restrictive assumption if marginal costs are independent of output levels, and markets are segmented, for then one may reasonably suppose that …rms play separate and independent Cournot games in separate markets.

Section 2 deals with the equilibrium pattern of taxes and subsidies in the "third market" model. In Section 3 we briefly look at the case of competition in the home market. Some concluding remarks are made in Section 4. Proofs are provided in the Appendix.

2 International rent diffusion in the 'third market' model

2.1 The model

In this section we generalize the 'third market' model of Brander and Spencer (1985) by introducing five elements of complexity: (i) in each country, firms are ex ante asymmetric in terms of marginal costs, (ii) taxes and subsidies are firm-specific, (iii) the marginal cost of public finance generally exceeds unity, (iv) firms are internationally owned, and (v) governments can subsidize firms that are located abroad[4].

Let $I = \{1, 2, \ldots, n\}$ and $J = \{n + 1, \ldots, n + n^*\}$ be the sets of indices for firms located in Country 1 and Country 2 respectively. These $n + n^*$ firms have constant marginal costs and compete as Cournot rivals in a third market, which is unrelated to the markets in Country 1 and Country 2.The demand function in the third market is $p = p(Z)$, where $Z = Q_I + Q_J$, and

$$Q_I = \sum_{i \in I} q_i \quad ; Q_J = \sum_{j \in J} q_j \quad .$$

Let $K = \{1, 2, \ldots, n + n^*\}$. Firm $k \in K$ has a constant marginal cost c_k^0. For each unit of its output sold in the third market, it receives a subsidy s_k from Country 1 and s_k^* from Country 2: (The asterisk in s_k^* denotes that it is the policy variable of Country 2). Since Country 1 (respectively, Country 2) cannot tax, but can subsidize, the firms located in the other country, we impose the constraint that $s_j \geq 0$ for all $j \in J$ (respectively $s_i^* \geq 0$ for all $i \in I$). On the other hand, there are no sign restrictions for s_i ($i \in I$) nor for $s_j^*(j \in J)$; thus, for example, a negative s_i $(i \in I)$ means that Country 1 taxes firm i's output. This is a very important element of asymmetry in the model, it reflects the relevance of political boundary.

[4] The first element of complexity is dealt with in Long and Soubeyran (1997) in a model where firm-specific taxes (or subsidies) are not permitted. The second element is studied in Hwang and Mai (1991). Neary (1995) takes into account the third element. While Lee (1990) allows international ownership, he does not permit governments to subsidize firms located abroad.

The net subsidy received by firm k per unit of output is

$$u_k = s_k + s_k^* \tag{1}$$

and its profit is

$$\pi_k = p(Z)q_k - c_k q_k$$

where

$$c_k = c_k^0 - u_k$$

Let α_k (respectively α_k^*) denote the portion of equities of firm $k \in K$ owned by residents of Country 1 (respectively, Country 2). The sum of profits earned by Country 1's residents is

$$\Pi = \sum_{i \in I} \alpha_i \pi_i + \sum_{j \in J} \alpha_j \pi_j \tag{2}$$

Similarly for residents of Country 2:

$$\Pi^* = \sum_{i \in I} \alpha_i^* \pi_i + \sum_{j \in J} \alpha_j^* \pi_j \tag{3}$$

where, of course, $\alpha_k \geq 0; \alpha_k^* \geq 0; \alpha_k + \alpha_k^* = 1$, for all $k \in K$. Country 1's total net payment of subsidies is

$$S_I + S_J = \sum_{i \in I} s_i q_i + \sum_{j \in J} s_j q_j$$

Similarly for Country 2:

$$S_J^* + S_J^* = \sum_{i \in I} s_i^* q_i + \sum_{j \in J} s_j^* q_j$$

Since we are dealing with the 'third market' model, the welfare of Country 1 is

$$W = \Pi - (S_I + S_J)\delta \tag{4}$$

where $\delta > 1$ is the marginal cost of public finance. Similarly, for Country 2:

$$W^* = \Pi^* - (S_I^* + S_J^*)\delta \tag{5}$$

2.2 The two-stage game

We consider the following two-stage game. In the first stage, the two governments set their firm-specific subsidy (or tax) rates. They do so non-cooperatively, and both anticipate that in the second stage firms will attain a Cournot equilibrium in the third market. In the second stage, firms take the (firm-specific) subsidy or tax rates as given, and compete à la Cournot. As usual, this game is solved backwards, beginning with the second stage. We now characterize the equilibrium of the second stage.

The first order conditions for a Cournot equilibrium where all firms produce[5] are

$$q_k p'(Z) + p(Z) = c_k^0 - u_k \quad ; k \in K \tag{6}$$

The second order conditions are satisfied if

$$2 - \phi_k E > 0 \quad ; k \in K \tag{7}$$

where $\phi_k \equiv q_k/Z$ is firm k's market share and E is the elasticity of the slope of the demand curve: $E \equiv -Zp''(Z)/P'(Z)$. Summing (6) over all firms gives

$$Zp'(Z) + (n + n^*)p(Z) = (n + n^*)c_K^0 - nu_I - n^*u_J \tag{8}$$

where c_K^0 is the mean marginal cost (not including subsidy/tax), and u_I (respectively u_J) is the mean net subsidy rate received by firms located in Country 1 (respectively, 2):

$$c_K^0 = \frac{1}{n + n^*} \sum_{k \in K} c_k^0 \quad ; u_I = \frac{1}{n} \sum_{i \in I} u_i \quad ; u_J = \frac{1}{n^*} \sum_{j \in J} u_j$$

Equation (8) uniquely determines the equilibrium output Z; provided the right-hand side of (8) is a decreasing function of Z: This condition is satisfied if for all $Z \geq 0$ the following inequality holds: $E < n + n^* + 1$: (This inequality is one of the familiar stability conditions, see Dixit (1986).) Thus we can write the equilibrium output as $Z = Z(nu_I + n^*u_J)$: Substituting this into (6), we can write firm k's equilibrium output as

$$q_k = q_k(u_k; nu_I + n^*u_J) = \frac{\hat{p} - c_k^0 + u_k}{[-\hat{p}']} \tag{9}$$

[5] We assume that all firms produce in an equilibrium. The analysis can be extended to the case where some firms shut down because their costs (including taxes) are too high.

where the hat indicates equilibrium values. We shall assume that this quantity is strictly positive. The equilibrium profit of each firm can then be calculated. (See the Appendix.)

We are now ready to analyze the first stage of the game. The government of Country 1 has as policy instruments a vector of firm-specific subsidy-cum-tax rates $s = (s_1; s_2; ...; s_{n+n^*})$ where the first n elements can be positive, negative, or zero, but the last n^* elements can only be positive or zero, because it cannot tax the output of firms located abroad. Similarly the government of Country 2 has the instrument vector $s^* = (s_1^*; s_2^*; ...; s_{n+n^*}^*)$ where the first n elements cannot be negative. These constraints are recorded below for ease of reference:

$$s_j \geq 0 \quad \forall j \in J \quad ; \quad s_i^* \geq 0 \quad \forall i \in I \tag{10}$$

Each government chooses the rates of subsidy-cum-tax to maximize its welfare function (see (4) and (5) above), taking as given the choice made by the other government. To save space we will not elaborate the methods of solving for the Nash equilibrium of this game between governments; the readers are referred to the Appendix, or, for a more general perspective, to Long and Soubeyran (1997b). In what follows we describe our results for this two-stage game, and give some intuitive explanation.

2.3 The benchmark case

Let us begin by considering very briefly the formulas for subsidies in the benchmark case where there is no cross ownership. In this case no firm is subsidized by a foreign government, so that $c_i = c_i^0 + s_i$ for $i \in I$ and $c_j = c_j^0 + s_j^*$. Let $c_I = (1/n)\sum_{i \in I} c_i$; $s_I = (1/n)\sum_{i \in I} s_i$, $m_i = p - c_i$; and $m_I = p - c_I$. Similar notations apply to Country 2. Country 1's welfare function in this case is $W = \Pi - \delta S_I$, and, as shown in the Appendix,

$$W = \sum_{i \in I} [m_i - \delta s_i] m_i \tag{11}$$

Suppose the Nash equilibrium for the game between the two government has been found, what can we say about the pattern of subsidies within the home country?

Given s_J^* and s_I the equilibrium price is determined, and is denoted by $\hat{p}$. Furthermore, the sum of the outputs of firms located in Country 1 is also determined by s_J^* and s_I, and is independent of the s_i (as long as all firms produce, and $\sum_{i\in I} s_i = ns_I$. Using (11) the equilibrium firm-specific subsidies can be shown (see the Appendix) to depend on their cost characteristics in the following manner:

$$s_i - s_I = \frac{\delta - 2}{2(\delta - 1)}\left[c'_i - c'_I\right] \tag{12}$$

To understand the implications of this formula, consider two cases, according to whether δ is smaller or greater than 2:

Case 1: $1 < \delta < 2$

This may be called the normal case, because in the public finance literature it is widely held that $1 < \delta < 2$. Then formula (12) tells us that lower cost firms receive higher subsidies per unit of output. This seems intuitively reasonable because we tend to think that, for a given sum of outputs by home firms, it would be more efficient that the lower cost firms are given incentives to produce relatively more.

Case 2: $\delta > 2$

In this case, the cost of public fund is very high, and formula (12) tells us that lower cost firms are given lower subsidies per unit of output. This may sound counter-intuitive, but upon reflection this is the right policy when $\delta > 2$: For any fixed total of domestic outputs, this policy implies a loss of production efficiency (as higher cost domestic firms are encouraged to produce more relative to lower cost domestic firms), but a gain in terms of economizing public fund: the higher cost firms get more subsidy per unit, but the numbers of units they produce are few.

Apart from the above subsidy rule, the following point is worth making. It can be shown that[6]

$$\Pi = \frac{n}{(-p')}\left[m_I^2 + Var(c_i)\right] \tag{13}$$

and

$$S_I = \frac{n}{(-p')}\left[np(c'_I - c_I) + nc_I^2 + nVar(c_i) - \sum_{i\in I} c'_i c_i\right] \tag{14}$$

where, using (12),

$$Var(c_i) \equiv (1/n)\sum_{i\in I}\left[c_i - c_I\right]^2 = (1/n)\left(\frac{2\delta}{3\delta - 2}\right)^2 \sum_{i\in I}\left[c'_i - c'_I\right]^2$$

[6] See Long and Soubeyran (1997).

denote the variance of the distribution of marginal costs. This shows that, for a given equilibrium price and given c_I, both Π and S_I increases with the variance of marginal costs.

2.4 A separability result in a complex policy game context

We now turn to the case where there is international cross ownership. Despite the complexity of our model, it is quite remarkable that certain simple characterisations of equilibrium subsidy rates are possible. The solution exhibits a separability property: each firm-specific subsidy rate in equilibrium consists of two components. The first one is idiosyncratic: it reflects the particular cost characteristic of the firm. The second component depends only on the average characteristic of all firms. Furthermore, the overall subsidy that a firm receives from both countries is under certain conditions independent of the ownership pattern. The first theorem clearly exhibits this independence property.

Theorem 1:(Independence of ownership pattern) At a Nash equilibrium of the the game between the governments, if the constraints (10) are not binding[7], then the net combined subsidy received by firm i (located in Country 1) from both governments is equal to the average net subsidy received by firms located in that country plus a term that increases with its cost disadvantage:

$$u_i - u_I = \gamma(c_i^0 - c_I^0); \quad 0 < \gamma \equiv \frac{2\delta - 2}{3\delta - 2} < 1 \tag{15}$$

where $c_I^0 \equiv (1/n)\sum_{i\in I} c_i^0$. Similarly, for $j \in J; u_j - u_J = \gamma(c_j^0 - c_J^0)$: Furthermore, the profit margin of firm $i \in I$ deviates from the average profit margin in Country 1 by an amount that decreases with its cost disadvantage:

$$m_i - m_I = (\gamma - 1)(c_i^0 - c_I^0) \quad ; \gamma - 1 < 0 \tag{16}$$

where $m_i \equiv \bar{p} - c_i^0 + u_i$ is firm i's profit margin and $m_I \equiv (1/n)\sum_{i\in I} m_i$. Similarly, for $j \in J$, $m_j - m_J = (\gamma - 1)(c_j^0 - c_J^0)$.

Remark: This theorem says that firms whose ex ante costs are higher than the average will receive a higher net subsidy than average, but their profits

[7] This means that the associated Lagrange multipliers are zero at the solution point.

per unit remain smaller than the average profit. In other words, the subsidies do not fully compensate for their initial cost disadvantages. The greater is the marginal cost of public finance δ, the greater is the weight given to the cost disadvantage. We can write (15) as

$$u_i = \gamma(c_i' - c_I') + u_I \tag{17}$$

This shows that the equilibrium net subsidy that firm i receives from the two governments consists of two components. The first one is idiosyncratic, as it depends on c_i', and the second component is common to all firms in the same country. It is remarkable that in these formulas that relate cost deviations to deviations of combined sudsidies, the ownership patterns plays no role. This independence property does not hold when we look at the subsidy received from a given country.

Theorem 2:(Dependence of a country's subsidy on ownership pattern) At a Nash equilibrium of the the game between the governments, if the constraints (10) are not binding, then the subsidy s_i (respectively s_i^*) received by firm $i \in I$ from Government 1 (respectively, Government 2) are

$$s_i = s_I + (c_i' - c_I')\left(\frac{\delta - 2\alpha_I}{3\delta - 2}\right) + \frac{2}{\delta}(\alpha_i - \alpha_I)m_I -$$
$$\frac{2}{3\delta - 2}\left[(\alpha_i - \alpha_I)(c_i' - c_I') - \mathrm{cov}(\alpha_i; c_i')\right] \tag{18}$$

and

$$s_i^* = s_I^* + (c_i' - c_I')\left(\frac{\delta - 2\alpha_I^*}{3\delta - 2}\right) + \frac{2}{\delta}(\alpha_i^* - \alpha_I^*)m_I -$$
$$\frac{2}{3\delta - 2}\left[(\alpha_i^* - \alpha_I^*)(c_i' - c_I') - \mathrm{cov}(\alpha_i^*; c_i')\right] \tag{19}$$

where $\mathrm{cov}(\alpha_i; c_i')$ is the covariance between domestic ownership and marginal cost :

$$\mathrm{cov}(\alpha_i; c_i') \equiv \frac{1}{n}\sum_{i \in I}(\alpha_i - \alpha_I)(c_i' - c_I'); \quad \alpha_I \equiv \frac{1}{n}\sum_{i \in I}\alpha_i$$

and $\mathrm{cov}(\alpha_i^*; c_i')$ is the covariance between foreign ownership and marginal cost :

$$\mathrm{cov}(\alpha_i^*; c_i') \equiv \frac{1}{n}\sum_{i \in I}(\alpha_i^* - \alpha_I^*)(c_i' - c_I'); \quad \alpha_I^* \equiv \frac{1}{n}\sum_{i \in I}\alpha_i^*$$

A similar result holds for firms in Country 2.

Proof : see the Appendix.

Notice that if we take the limit as α_i and α_I approach 1, the formula (18) does not converge to (12). This is because (18) is obtained under the assumption that each government gives positive subsidies to firms located abroad, and this is the case only if α_i is strictly bounded away from 1.

Remark: The equilibrium deviation of the subsidy s_i from the average s_I can be written more compactly as follows:

$$s_i - s_I = (c'_i - c'_I)\left(\frac{\delta - 2\alpha_i}{3\delta - 2}\right) + \frac{2}{\delta}(\alpha_i - \alpha_I)m_I + \frac{2}{3\delta - 2}\mathrm{cov}(\alpha_i; c'_i) \quad (20)$$

From this formula, three components emerge. The first one, $d_i^A = (c'_i - c'_I)\frac{\delta - 2\alpha_i}{3\delta - 2}$ shows that if a domestically located firm is predominently owned by domestic residents and the cost of public finance is not too great (ie, $\delta - 2\alpha_i < 0$), then the lower is its cost, the higher will be the subsidy it receives from the home government. If its cost is smaller than average by $\epsilon > 0$ then its subsidy is greater than average by $(2\alpha_i - \delta)\epsilon/(3\delta - 2)$. This factor is an increasing function of the extent of domestic ownership. Conversely, if $\delta > 2\alpha_i$ then the lower is its cost, the smaller will be the subsidy. These results are in broad agreement with the benchmark case in the preceding subsection. The second term, $d_i^B = \frac{2}{\delta}(\alpha_i - \alpha_I)m_I$ is independent of cost disadvantage. It relates only to the relative position of the firm in terms of ownership by domestic residents. The greater the proportion of domestic onwership of firm i relative to that of other firms, the higher is the subsidy bestowed on it. The third term, $d_i^C = \frac{2}{3\delta - 2}\mathrm{cov}(\alpha_i; c'_i)$ increases with the covariance of the domestic ownership with the marginal cost. Thus, for example, if firm i is a high cost firm and if within Country 1 the proportion of domestic ownership tends to be higher for higher cost firms, then this term implies that firm i will tend to be favoured relative to other firms in the same country.

Concerning the subsidies that firms located in Country 1 receive from the foreign country, the deviation from the mean can be expressed as follows:

$$s_i^* - s_I^* = (c'_i - c'_I)\left(\frac{\delta - 2\alpha_i^*}{3\delta - 2}\right) + \frac{2}{\delta}(\alpha_i^* - \alpha_I^*)m_I + \frac{2}{3\delta - 2}\mathrm{cov}(\alpha_i^*; c'_i) \quad (21)$$

Two particular cases are worth mentioning. Suppose that all firms located in Country 1 have identical marginal costs. Then from (20) and (21), we have

$s_i - s_I = -(s_i^* - s_I^*)$. The deviations are in opposite directions. The second special case of interest is when firms in Country 1 are equally owned by residents of the two countries. Then $s_i - s_I = s_i^* - s_I^*$, that is, the deviations are the same.

Comparing (20) with the benchmark rule (12), we see that the 'distortion' created by international cross ownership may be represented as follows

$$(s_i - s_I) - \frac{\delta - 2}{2(\delta - 1)}(c_i^0 - c_I^0) = f(\alpha_i; \delta)(c_i^0 - c_I^0) +$$

$$\frac{2}{\delta}(\alpha_i - \alpha_I)m_I + \frac{2}{3\delta - 2}\operatorname{cov}(\alpha_i; c_i^0) \tag{22}$$

where

$$f(\alpha_i; \delta) \equiv \frac{\delta - 2\alpha_i}{3\delta - 2} - \frac{\delta - 2}{2(\delta - 1)}$$

This term is positive if α_i is small.

2.5 Corner solutions

Consider now the case in which, at the equilibrium of the game between the two governments, no government subsidizes firms that are located abroad (in fact, at this equilibrium each government would wish to have the power to tax firms that are not located in its country). Intuitively, we expect this to happen if firms located in the foreign country are owned by residents of that country. The following theorem characterizes this type of equilibrium.

Theorem 3: In an equilibrium where no government subsidizes firms that are located abroad, that is, $s_i^* = s_j = 0$, for all $i \in I$ and all $j \in J$, the equilibrium subsidies are

$$s_i - s_I = \left[\rho_I - a_i c_i^0\right] + (b_i - b_I)z_I + (a_i - a_I)\mathbf{p} \tag{23}$$

$$s_j^* - s_J^* = \left[\rho_J^* - a_j c_{i_j}^0\right] + \left(b_j^* - b_J^*\right)z_J^* + (a_j^* - a_J^*)\mathbf{p} \tag{24}$$

where

$$a_i \equiv \frac{\delta - 2\alpha_i}{2(\alpha_i - \delta)} \quad ; a_j^* \equiv \frac{\delta - 2\alpha_j^*}{2(\alpha_j^* - \delta)} \quad ; b_i \equiv \frac{\delta}{2(\alpha_i - \delta)} \quad ; b_j^* \equiv \frac{\delta}{2(\alpha_j^* - \delta)}$$

$$a_I \equiv \frac{1}{n}\sum_{i\in I} a_i \quad ; a_J^* \equiv \frac{1}{n^*}\sum_{j\in J} a_j^* \quad ; b_I \equiv \frac{1}{n}\sum_{i\in I} b_i \quad ; b_J^* \equiv \frac{1}{n^*}\sum_{j\in J} b_j^*$$

and

$$\rho_I \equiv \frac{1}{n}\sum_{i\in I} a_i c_i^0; \quad \rho_J^* \equiv \frac{1}{n^*}\sum_{j\in J} a_j^* c_j^0$$

$$z_I \equiv \frac{1}{b_I}[s_I + \rho_I - \mu a_I]; \quad z_J^* \equiv \frac{1}{b_J^*}[s_J^* + \rho_J^* - \mu a_J^*]$$

Proof :see the Appendix.

Remark: Let us concentrate on the subsidy (or tax) set by Country 1 for its local firm i. The deviation s_i from the mean s_I involves two components. First, $d_i^D = \frac{1}{n}\sum_{i\in I} a_i c_i^0 - a_i c_i^0$ reflects the cost heterogeneity of the local firms, weighted by the extent of foreign ownership. It contains the idiosyncratic cost term c_i^0, just as did the term d_i^A in the preceding sub-section. However, unlike d_i^A, this term involves not just α_i but also all other $\alpha_{i'}$ where $i' \in I$. The second component, $d_i^E = (b_i - b_I) z_I + (a_i - a_I)\mu$, does not directly involve the idiosyncratic cost term c_i^0; but only indirectly as a component of industry average. The ownership profile is reflected in this term, but not in the form of a covariance expression. Two special cases are worth mentioning:

(i) If each local firm is 50% owned by domestic residents, then the subsidies are inversely related to relative cost disadvantages:

$$s_i - s_I = \frac{\delta - 1}{1 - 2\delta}(c_i^0 - c_I^0)$$

(ii) If all local firms have the same marginal cost, then

$$s_i - s_I = (\mu - c_I^0)(a_i - a_I) + (b_i - b_I)z_I$$

2.6 Neutrality results

In a model where firms compete in Country 2's market rather than in a third market, and with only one firm in each country, Lee (1990) obtains the neutrality result that that the equilibrium pattern of trade, the profit of the firms, and world welfare are independent of the ownership pattern. We will turn to this model in the next section. In the present sub-section, we study the possibility of obtaining a similar neutrality result in the third market model when there are heterogenous firms (both in tems of cost and in terms of ownership) in each country. A key point is that each government can subsidize any firm anywhere, but due to the third market assumption, no government can tax firms that are located abroad. This constraint was stated as (10). The answer turns out to depend on the nature of the Nash equilibrium in the game between the two government. If at the equilibrium the constraint (10) is not binding and in each country each firm receives a subsidy from the other country, then the neutrality result holds. On the other hand, if the constraint binds, so that at least one government would wish that the constraint is relaxed for at least one foreign firm, then in general non-neutrality obtains.

To prove neutrality in our complex model, we must at first prove a number of lemmas that are also of interest.

Lemma 1: Equilibrium price and output depend only on the sum $nu_I + n^*u_J$ and is independent of the pattern of ownership.

This lemma follows from (8). We denote the equilibrium output and price by $\hat{Z}$ and $\hat{p}$ respectively.

Lemma 2: The equilibrium pattern of trade in this two-stage game depends only on u_I and u_J and is independent of the pattern of ownership. In particular, firm i's equilibrium output is

$$\hat{q}_i = \frac{\hat{p} - c_i^0 + u_I + \gamma(c_i^0 - c_I^0)}{-p'(\hat{Z})} \quad ; i \in I; \quad 0 < \gamma = \frac{2\delta - 2}{3\delta - 2} < 1 \tag{25}$$

A similar expression holds for firm $j \in J$.

Proof: Use (9) and equation (51) in the Appendix.

Lemma 3: The equilibrium profit of each firm in this two-stage game depends only on u_I and u_J and is independent of the pattern of ownership. Specifically, firm i's equilibrium profit is

$$\hat{\pi}_i = \frac{1}{-p'(\hat{Z})}\left[\hat{m}_I + (\gamma - 1)(c_i^0 - c_I^0)\right]^2 \quad ; i \in I \tag{26}$$

where $\hat{m}_I \equiv \hat{p} - c_I^0 + u_I$: A similar expression holds for $j \in J$.

Proof: See the Appendix.

Lemma 4: The equilibrium aggregate profit earned by residents of Country 1 is

$$\hat{\Pi}_I = \frac{1}{-p'(\hat{Z})}\left[n\alpha_I \hat{m}_I^2 + n^*\alpha_J \hat{m}_J^2 + (\gamma - 1)^2\left(nV_\alpha(c_i^0) + n^*V_\alpha(c_j^0)\right)\right] \tag{27}$$

where $V_\alpha(c_i^0)$ is the modified variance of marginal costs weighted by the domestic ownership profile $(\alpha_1;\ldots;\alpha_n)$:

$$V_\alpha(c_i^0) \equiv \frac{1}{n}\sum_{i\in I}\alpha_i(c_i^0 - c_I^0)^2$$

and

$$V_\alpha(c_j^0) \equiv \frac{1}{n^*}\sum_{j\in J}\alpha_j(c_j^0 - c_J^0)^2$$

Similarly, the equilibrium aggregate profit earned by residents of Country 2 is

$$\Pi_I^* = \frac{1}{-p'(Z)}\left[n\alpha_I^* m_I^2 + n^*\alpha_J^* m_J^2 + (\gamma-1)^2(nV_{\alpha^*}^*(c_i^0) + n^*V_{\alpha^*}^*(c_j^0))\right]$$

where the definitions of $V_{\alpha^*}^*(c_i^0)$ and $V_{\alpha^*}^*(c_j^0)$ are obvious.

Proof : See the Appendix.

Remark: Lemma 1 is a striking result: The total profit earned by residents of a country depends on the modified variances of the distribution of marginal costs in the two countries[8]. In fact, we can link total profit with the Herfindahl index of industry concentration. Thus, let us define the modified Herfindahl index of the firms located in Country 1 as

$$H_{I\alpha} = \sum_{i\in I}\alpha_i\left(\frac{q_i}{Z}\right)^2$$

then it can be shown that this measure of concentration is related to the modified variance in a simple way

$$H_{I\alpha} = \left[\frac{1}{-2p'}\right]^2\left[n\alpha_I m_I^2 + 2\gamma n(1 + m_I - u_I)\mathrm{cov}(\alpha_i;c_i^0) + n\gamma^2 V_\alpha(c_i^0)\right]$$

For given u_I and u_J the modified Herfindahl index is greater, the greater is the modified variance $V_\alpha(c_i^0)$.

Finally, to prove the neutrality result, one must show that the equilibrium average subsidies u_I and u_J are independent of the ownership structure. Effectively, this means that if there is an exogenous change in the ownership

[8] This is a generalisation of the results obtained in Long and Soubeyran (1997) where there is no cross ownership.

pattern (for example, some Hongkong residents become Canadian residents, bringing with them their equities in Hongkong …rms), then while the subsidies given by the two governments will change, the net change in subsidy received by any …rm will be zero. This is an intuitively appealing result.

Theorem 4: In a Nash equilibrium of the game between the two governments, if the constraints (10) are not binding, and $s_i^{¤} > 0$ for all $i\ 2\ I$, $s_j > 0$ for all $j\ 2\ J$, then the following quantities are independent of the ownership pro…le: (i) the net subsidy received by each …rm, (ii) the output of each …rm, (iii) the pattern of trade, (iv) the value of each …rm, and (v) world welfare.
Proof : See the Appendix
Remark: The expression for equilibrium world welfare, as displayed in the Appendix, is very simple. It is independent of the ownership pro…le, but it does depend on the variance of the marginal costs of …rms.

2.7 Non-neutrality results: corner solutions

In the case of corner solutions, the neutrality result does not hold. This is related to the non-neutrality result in the literature on voluntary contributions to a public good, see the one-stage game model of Bergstrom et al. (1986), where a redistribution of income does a¤ect the quantity of public good, if some agents do not contribute in equilibrium. Starting at a corner solution, a change in income distribution will generally change their contribution status. However our model is a two-stage game and displays greater complexity in the speci…cation of income.

3 International rent di¤usion in a home market model

In this section we brie‡y indicate how the third market model analyzed above di¤ers from a version of the so-called "home market" model where …rms from the two countries compete in the market of one country, say Country 1. This is a generalised version of Lee's model (1990), which was built on Brander-Krugman (1983) and Dixit (1984). The model is almost identical to the third market model that we examined in detail in Section 2, except for the following:

(i) the market in which the …rms compete is not a third market; instead it is the 'home' market. The …rms $i\ 2\ I$ are located in the home country

(Country 1) and are called the home firms for convenience (even though they may be owned by residents of the foreign country).

(ii) we allow the foreign government to subsidize the home firms, but it cannot tax them. That is, s_i^* can be positive but cannot be negative. Lee (1990) did not allow such cross-border subsidies (s_i^* is required to be identically zero in his model.)

(iii) the home country can now tax or subsidize firms $j \in J$ because the taxes can be interpreted as trade taxes. Thus s_j is now unrestricted in sign. (In Section 2, because j sells in a third market, we did not allow s_j to be negative.)

(iv) the consumers'surplus in the home country is part of home welfare. Thus we must add a term $S(Z)$ to represent this component of welfare.

Notice that in this model, as in the third market model, we allow the home government to tax or subsidize home firms (s_i is unrestricted in signs). Lee (1990) did not allow the home government to interfere with home firms (s_i is required to be identically zero in his model.) The methods of analysis we use in this paper can be applied to this "home market" model.

4 Conclusion

International rent diffusion due to portfolio diversification in a global equity market renders extremely complex the task of any government that wishes to use strategic trade policies to improve the welfare of its nationals. In a model of game between two governments, we have been able to characterize the equilibrium taxes and subsidies, allowing for any ownership profile, cost heterogeneity, and a non-linear demand function. The method developed in Long and Soubeyran (1997) allows us to analyze this game and obtain some clear results despite the complexity of the model. This study is in the same spirit as that of Krueger and Duncan (1993) who examine the problem of regulation in a complex world.

There are a number of feasible extensions. Increasing or decreasing return to scale is a worthwhile direction of generalization. Uncertainty, which is the reason for portfolio diversification, could be introduced. It would also be interesting to consider a game involving several governments.

APPENDIX

The second stage: Cournot equilibrium

The first order conditions for a Cournot equilibrium are

$$q_k p'(Z) + p(Z) = c_k^0 - u_k \equiv c_k \quad ; k \in K \tag{28}$$

Summing over all firms to obtain

$$Zp'(Z) + (n + n^*)p(Z) = C^0 - nu_I - n^*u_J \quad ; C^0 \equiv \sum_{k \in K} c_k^0 \tag{29}$$

Therefore the equilibrium Z depends only on $nu_I + n^*u_J$ and not on the distribution of s_k. The equilibrium quantities and profits are

$$\hat{q}_k = \frac{\hat{p} - c_k}{(-\hat{p}')} \equiv \frac{\hat{m}_k}{(-\hat{p}')} \quad ; \hat{\pi}_k = \frac{(\hat{p} - c_k)^2}{(-\hat{p}')} \equiv \frac{\hat{m}_k^2}{(-\hat{p}')} \tag{30}$$

where $\hat{m}_k$ is firm k's equilibrium profit margin

$$\hat{m}_k = \hat{p} - c_k = \hat{p} - c_k^0 + u_k \equiv \hat{m}_k^0 + u_k \tag{31}$$

The welfare levels are

$$W = \frac{1}{(-\hat{p}')} \left[\sum_{i \in I} \hat{m}_i (\alpha_i \hat{m}_i - \delta s_i) + \sum_{j \in J} \hat{m}_j (\alpha_j \hat{m}_j - \delta s_j) \right] \tag{32}$$

$$W^* = \frac{1}{(-\hat{p}')} \left[\sum_{i \in I} \hat{m}_i (\alpha_i^* \hat{m}_i - \delta s_i^*) + \sum_{j \in J} \hat{m}_j \left(\alpha_j^* \hat{m}_j - \delta s_j^* \right) \right] \tag{33}$$

The first stage: Nash equilibrium of the subsidy-tax game

In stage 1, the two governments choose the vectors s and s^* non-cooperatively and simultaneously. To characterize the Nash equilibrium of this game, it is convenient to proceed as though the governments solve their problem in two steps. First, they take as given the average rates of subsidies $s_I; s_J; s_I^*$ and s_J^* and Country 1 (respectively 2) maximizes W (resp. W^*) by choosing the vector s (resp. s^*) subject to the equality constraints $\sum_{i \in I} s_i = ns_I$ and $\sum_{j \in J} s_j = n^*s_J$ and $s_j \geq 0$ for $j \in J$ (resp. $\sum_{i \in I} s_i^* = ns_I^*$ and $\sum_{j \in J} s_j^* = n^*s_J^*$ and $s_i^* \geq 0$ for $i \in I$). The chosen subsidy rates can therefore be expressed as functions of $s_I; s_J; s_I^*$ and s_J^* . In the second steps, $s_I; s_J; s_I^*$ and s_J^* are chosen.

The first step

Given $s_I; s_J; s_I^*$ and s_J^* , consider the Lagrangian for Country 1:

$$L = W + \lambda_I[ns_I - \sum_{i \in I} s_i] + \lambda_J[n^* s_J - \sum_{j \in J} s_j] + \sum_{j \in J} \mu_j s_j \tag{34}$$

With a suitable transformation of variables (as shown below), the first order conditions derived from (34) can be written compactly as follows

$$(2\beta_i - 1)\pi_i - s_i - z_I = 0 \tag{35}$$

$$(2\beta_j - 1)\pi_j - s_j - z_J + \nu_j = 0 \tag{36}$$

$$\nu_j \geq 0; \quad \nu_j s_j = 0 \tag{37}$$

where $\beta_i \equiv \alpha_i/\delta$, $\beta_j \equiv \alpha_j/\delta$, $z_I \equiv (-p')\lambda_I/\delta$, $z_J \equiv (-p')\lambda_J/\delta$, $\nu_j \equiv (-p')\mu_j/\delta$.

Remark: for the bechmark case, we obmit the last two terms in (34) and obtain from (35) the equation (12).

Similarly, for Country 2,

$$(2\beta_i^* - 1)\pi_i - s_i^* - z_I^* + \nu_i^* = 0 \tag{38}$$

$$(2\beta_j^* - 1)\pi_j - s_j^* - z_J^* = 0 \tag{39}$$

$$\nu_i^* \geq 0; \quad \nu_i^* s_i = 0 \tag{40}$$

From (35), (36), (38) and (39) we have

$$\pi_i - \pi_i^0 = u_i = s_i + s_i^* = 2[\beta_i + \beta_i^* - 1]\pi_i - z_I - z_I^* + \nu_i^*$$

which yields

$$\pi_i = \frac{z_I + z_I^* - \nu_i^* - \pi_i^0}{2[\beta_i + \beta_i^*] - 3} \tag{41}$$

Similarly

$$\pi_j = \frac{z_J + z_J^* - \nu_j - \pi_j^0}{2[\beta_j + \beta_j^*] - 3} \tag{42}$$

Substituting (41) and (42) into (35), (36), (38) and (39), we have

$$s_i = (2\beta_i - 1)\pi_i - z_I = \alpha_i \left[z_I + z_I^* - \nu_i^* - \pi_i^0 \right] - z_I \tag{43}$$

$$s_j = (2\beta_j - 1)\pi_j - z_J + \nu_j = \alpha_j \left[z_I + z_I^* - \nu_j - \pi_j^0 \right] - z_J + \nu_j \tag{44}$$

$$s_i^* = (2\beta_i^* - 1)\pi_i - z_I^* + v_i^* = \alpha_i^* \left[z_I + z_I^* - \nu_i^* - \pi_i^0 \right] - z_I^* + \nu_i^* \tag{45}$$

$$s_j^* = (2\beta_j^* - 1)\pi_j - z_J^* = \alpha_j^* \left[z_I + z_I^* - \nu_j - \pi_j^0 \right] - z_J \tag{46}$$

where $\alpha_i \equiv [2\beta_i - 1]/[2(\beta_i + \beta_i^*) - 3]$, $\alpha_i^* \equiv [2\beta_i^* - 1]/[2(\beta_i + \beta_i^*) - 3]$, $\alpha_j \equiv [2\beta_j - 1]/[2(\beta_j + \beta_j^*) - 3]$; $\alpha_j^* \equiv [2\beta_j^* - 1]/[2(\beta_j + \beta_j^*) - 3]$. It follows that

$$u_i = s_i + s_i^* = (\gamma - 1)(z_I + z_I^*) - \gamma(\nu_i^* + \pi_i^0) + \nu_i^* \tag{47}$$

$$u_j = s_j + s_j^* = (\gamma - 1)(z_j + z_J^*) - \gamma(\nu_j + \pi_j^0) + \nu_j \tag{48}$$

where $0 < \gamma \equiv [2\delta - 2]/[3\delta - 2] < 1$: Summing (47) over all $i \in I$ and (48) over all $j \in J$, we obtain

$$nu_I = n(\gamma - 1)(z_I + z_I^*) - \sum_{i \in I} \gamma(\nu_i^* + \pi_i^0) + \sum_{i \in I} \nu_i^* \tag{49}$$

$$n^* u_J = n^*(\gamma - 1)(z_J + z_J^*) - \sum_{j \in J} \gamma(\nu_j + \pi_i^0) + \sum_{j \in J} \nu_j \tag{50}$$

Proof of Theorem 1

When the constraints (10) are not binding, $\nu_j = 0$ for all $j \in J$ and $\nu_i^* = 0$ for all $i \in I$. Then from (47) and (49)

$$u_i - u_I = \gamma(c_i^0 - c_I^0); \quad i \in I \tag{51}$$

Similarly

$$u_j - u_J = \gamma(c_j^0 - c_J^0); \quad j \in J \tag{52}$$

Now since $\pi_i = \pi_i^0 + u_i$, (51) gives

$$\pi_i = \pi_i^0 + u_I + \gamma(c_i^0 - c_I^0); \quad i \in I \tag{53}$$

Similarly

$$\bar{c}_j = \bar{c}_j^0 + u_J + \gamma(c_j^0 - c_J^0); \quad j \in J \tag{54}$$

Let us define

$$\bar{c}_I \equiv a - c_I^0 + u_I; \quad \bar{c}_J \equiv a - c_J^0 + u_J \tag{55}$$

Then, using (53) and (54),

$$\bar{c}_i = \bar{c}_I + (\gamma - 1)(c_i^0 - c_I^0); \quad i \in I \tag{56}$$

$$\bar{c}_j = \bar{c}_J + (\gamma - 1)(c_j^0 - c_J^0); \quad j \in J \tag{57}$$

Proof of Theorem 2

From (43) -(46) and (56)-(57) we get

$$s_i = (2\beta_i - 1)\left[\bar{c}_I + (\gamma - 1)(c_i^0 - c_I^0)\right] - z_I \tag{58}$$

$$s_i^* = (2\beta_i^* - 1)\left[\bar{c}_I + (\gamma - 1)(c_i^0 - c_I^0)\right] - z_I^* \tag{59}$$

$$s_j = (2\beta_j - 1)\left[\bar{c}_J + (\gamma - 1)(c_j^0 - c_J^0)\right] - z_J \tag{60}$$

$$s_j^* = (2\beta_j^* - 1)\left[\bar{c}_J + (\gamma - 1)(c_j^0 - c_J^0)\right] - z_J^* \tag{61}$$

Summing over $i \in I$ or $j \in J$, then dividing by n or n^*, we obtain

$$s_I = (2\beta_I - 1)\bar{c}_I + 2(\gamma - 1)\mathrm{cov}(\beta_i; c_i^0) - z_I \tag{62}$$

$$s_I^* = (2\beta_I^* - 1)\bar{c}_I + 2(\gamma - 1)\mathrm{cov}(\beta_i^*; c_i^0) - z_I^* \tag{63}$$

$$s_J = (2\beta_J - 1)\bar{c}_J + 2(\gamma - 1)\mathrm{cov}(\beta_j; c_j^0) - z_J \tag{64}$$

$$s_J^* = (2\beta_J^* - 1)\bar{c}_J + 2(\gamma - 1)\mathrm{cov}(\beta_j^*; c_j^0) - z_J^* \tag{65}$$

These four equations show that the four variables $z_I; z_J; z_I^*$ and z_J^* are linear functions of $s_I; s_J; s_I^*$ and s_J^* (Recall that from (55) $\bar{c}_I$ and $\bar{c}_J$ are linear functions of these four variables). Re-write (58) as

$$s_i = (2\beta_i - 1)\bar{c}_I + (\gamma - 1)(c_i^0 - c_I^0)\left[2(\beta_i - \beta_I) + (2\beta_i - 1)\right] - z_I \tag{66}$$

From (62) and (66) we obtain (18), as was to be proved.

Proof of Theorem 4

So far, we have taken $s_I; s_J; s_I^*$ and s_J^* as given and expressed all other variables as functions of these four basic variables. We can now express the welfare of each country in terms of these four variables, and then obtain the necessary conditions that characterize their values in equilibrium. After some tedious manipulation, we obtain:

$$\widehat{W} = \frac{1}{[-p'(\hat{Z})]}[A + B]; \quad \widehat{W}^* = \frac{1}{[-p'(\hat{Z})]}[A^* + B^*] \tag{67}$$

where

$$A \equiv 2(1 - \gamma)\left[\widehat{m}_I \operatorname{cov}(\alpha_i; c_i^0) + \widehat{m}_J \operatorname{cov}(\alpha_j; c_j^0)\right] \tag{68}$$

and

$$\begin{aligned} B = {} & n(\delta - \alpha_I)\left[\widehat{m}_I^2 + (\gamma - 1)Var(c_i^0)\right] + n^*(\delta - \alpha_J)\left[\widehat{m}_J^2 + (\gamma - 1)Var(c_j^0)\right] \\ & - (\gamma - 1)^2 \sum_{i \in I} (\alpha_i - \alpha_I)(c_i^0 - c_I^0)^2 - (\gamma - 1)^2 \sum_{j \in J} (\alpha_j - \alpha_J)(c_j^0 - c_J^0)^2 \\ & + \delta n z_I \widehat{m}_I + \delta n^* z_J \widehat{m}_J \end{aligned} \tag{69}$$

Similarly,

$$A^* \equiv 2(1 - \gamma)\left[\widehat{m}_I \operatorname{cov}(\alpha_i^*; c_i^0) + \widehat{m}_J \operatorname{cov}(\alpha_j^*; c_j^0)\right] \tag{70}$$

$$\begin{aligned} B^* = {} & n(\delta - \alpha_I^*)\left[\widehat{m}_I^2 + (\gamma - 1)Var(c_i^0)\right] + n^*(\delta - \alpha_J^*)\left[\widehat{m}_J^2 + (\gamma - 1)Var(c_j^0)\right] \\ & - (\gamma - 1)^2 \sum_{i \in I} (\alpha_i^* - \alpha_I^*)(c_i^0 - c_I^0)^2 - (\gamma - 1)^2 \sum_{j \in J} (\alpha_j^* - \alpha_J^*)(c_j^0 - c_J^0)^2 \\ & + \delta n z_I^* \widehat{m}_I + \delta n^* z_J^* \widehat{m}_J \end{aligned} \tag{71}$$

where $z_I; z_J; z_I^*$ and z_J^* are linear functions of $s_I; s_J; s_I^*$ and s_J^*, see (62)-(65). Observe that

$$A + A^* = 0 \tag{72}$$

and $B + B^*$ is independent of the pattern of cross ownership:

$$B + B^* = (2\delta - 1)\left[n\widehat{m}_I^2 + n^*\widehat{m}_J^2 + n(\gamma - 1)Var(c_i^0) + n^*(\gamma - 1)Var(c_j^0)\right] +$$

$$n\bar{x}_I[2\bar{x}_I(1-\delta)-\delta u_I]+n^*\bar{x}_J[2\bar{x}_J(1-\delta)-\delta u_J] \quad (73)$$

(This follows from the facts that, from (62)-(65),

$$z_I+z_I^*=-u_I+2\bar{x}_I(1-\delta)/\delta \quad (74)$$

and similarly for J, and $\mathrm{cov}(\alpha_i+\alpha_i^*, c_i^0)=0$, also for j). This proves that world welfare is dependent only on the net average subsidies u_I and u_J. It remains to show that u_I and u_J are independent of the ownership profile (even though s_I, s_J, s_I^* and s_J^* are not independent of the ownership profile). From (67)

$$\frac{\partial \tilde{W}^*}{\partial s_I^*}=\frac{\partial \tilde{W}^*}{\partial s_I}-\delta n\bar{x}_I;\quad \frac{\partial \tilde{W}^*}{\partial s_J^*}=\frac{\partial \tilde{W}^*}{\partial s_J}-\delta n^*\bar{x}_J \quad (75)$$

Using (77) and the first order conditions

$$\frac{\partial \tilde{W}^*}{\partial s_I^*}=0;\quad \frac{\partial \tilde{W}^*}{\partial s_J^*}=0;\quad \frac{\partial \tilde{W}}{\partial s_I}=0;\quad \frac{\partial \tilde{W}}{\partial s_J}=0 \quad (76)$$

we obtain

$$\frac{\partial \tilde{W}}{\partial s_I}+\frac{\partial \tilde{W}^*}{\partial s_I}-\delta n\bar{x}_I=0;\frac{\partial \tilde{W}}{\partial s_J}+\frac{\partial \tilde{W}^*}{\partial s_J}-\delta n^*\bar{x}_J=0 \quad (77)$$

Now re-write (77) as

$$\frac{\partial}{\partial u_I}\left[\tilde{W}+\tilde{W}^*\right]-\delta n\bar{x}_I=0;\frac{\partial}{\partial u_J}\left[\tilde{W}+\tilde{W}^*\right]-\delta n^*\bar{x}_J=0 \quad (78)$$

The two equations in (78) determine the equilibrium u_I and u_J independently of the ownership profile, because $\tilde{W}+\tilde{W}^*$ is independent of the ownership profile.

Proof of Theorem 3

We now turn to the case where at the Nash equilibrium no government subsidizes firms located abroad, that is $s_i^*=s_j=0$ for all $i\in I$ and $j\in J$. Then we have (35), (39), and $\bar{x}_i-\bar{x}_i^0=s_i$, $\bar{x}_j-\bar{x}_j^0=s_j$, and from these four equations, we obtain

$$\bar{x}_i=b_i[z_I-\bar{x}_i^0];\quad \bar{x}_j=b_j^*[z_J^*-\bar{x}_j^0] \quad (79)$$

$$s_i=a_i\bar{x}_i^0+b_iz_I \quad (80)$$

$$s_j^* = a_j^* \hat{c}_j^0 + b_j^* z_J^* \qquad (81)$$

where $a_i \equiv [1 - 2\beta_i]/[2\beta_i - 2]$, $a_j^* \equiv [1 - 2\beta_j^*]/[2\beta_j^* - 2]$, $b_i \equiv 1/[2\beta_i - 2]$; $b_j^* \equiv 1/[2\beta_j^* - 2]$.

Summing over $i \in I$; $j \in J$ gives

$$s_I = a_i \hat{p} - \rho_I + z_I b_I; \quad s_J^* = a_j^* \hat{p} - \rho_J^* + z_J^* b_J^*$$

where $b_I \equiv (1/n)\sum_{i \in I} b_i$, $b_J^* \equiv (1/n^*)\sum_{j \in J} b_j^*$, $\rho_I \equiv (1/n)\sum_{i \in I} a_i c_i^0$, $\rho_J^* \equiv (1/n^*)\sum_{j \in J} a_j^* c_i^0$. Then

$$z_I = \frac{1}{b_I}[s_I + \rho_I - \hat{p} a_I]; \quad z_J^* = \frac{1}{b_J^*}[s_J^* + \rho_j^* - \hat{p} a_J^*] \qquad (82)$$

$$s_i - s_I = \hat{p}[a_i - a_I] - [a_i c_i - \rho_I] + [b_i - b_I] z_I \qquad (83)$$

where z_I is given in (82). The welfare of Country 1 is

$$\tilde{W} = \frac{1}{p'(\hat{Z})}\left[\sum_{i \in I} d_i [z_I - \hat{c}_i^0]^2 + \sum_{j \in J} e_j [z_J^* - \hat{c}_j^0]^2 + \delta n z_I \hat{c}_I\right] \qquad (84)$$

where $d_i \equiv [\delta - \alpha_i]/[4(\beta_i - 1)^2]$, $e_j \equiv \alpha_j/[4(\beta_j^* - 1)^2]$. Similarly

$$\tilde{W}^* = \frac{1}{p'(\hat{Z})}\left[\sum_{j \in J} d_j^* [z_J - \hat{c}_j^0]^2 + \sum_{i \in I} e_i^* [z_I - \hat{c}_i^0]^2 + \delta n^* z_J^* \hat{c}_J\right] \qquad (85)$$

where $d_j^* \equiv [\delta - \alpha_j^*]/[4(\beta_j^* - 1)^2]$, $e_i^* \equiv \alpha_i^*/[4(\beta_i - 1)^2]$. Clearly, the neutrality result does not hold in this case. (Numerical examples can be constructed to support this non-neutrality result.)

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